

TOPIC 7
**TAX INCIDENCE
& OPTIMALITY**



MICHAEL SACHS

Adjunct Professor | Southern Centre for Inequality Studies
University of the Witwatersrand | Johannesburg

- This week
 - Partial equilibrium
 - Incidence
 - Efficiency and deadweight loss
 - Commodity taxes
- Next week
 - General equilibrium
 - Income taxes (on labour and capital)
 - Equity and progressivity

- Institute for Fiscal Studies (2011) Tax By Design: The Mirrlees Review
 - Chapter 2 The Economic Approach to Tax Design
- Stiglitz, J. E. and Rosengard, J. K. (2015) Economics of the Public Sector
 - Chapter 18 Tax incidence
 - Chapter 19 Taxation and economic efficiency
 - Chapter 20 Optimal taxation

But also ...

 - Chapter 21 Taxation of capital
 - Chapter 22: The personal income tax
- Hindriks, J. and Myles, G. D. (2013) Intermediate Public Economics
 - Chapter 15 Commodity taxation
 - Chapter 16 Income Taxation
- Also: Black, P. A., Calitz, E., Steenekamp, T. J. and Black, P. A. (2015) Public Economics
 - Chapters 11, 12, 13 and 14
 - Good coverage of incidence and efficiency, less on optimal taxation

Preliminary issues in taxation

- The tax system is “enormous and fearsomely complex”
- For a given distributional outcome the tax system should aim for:
 - Minimize negative effects on welfare and economic efficiency
 - Minimize compliance costs
 - Be procedurally fair and non-discriminatory
 - Transparency
- This is achieved by a simple, neutral and stable tax system
 - Neutrality: The same activities attract the same rate of taxation
 - Simplicity: complexity and avoidance dynamic
 - Stability: compliance costs and long term planning
- The trade-off between redistribution and efficiency is at the centre of debates about the tax system.
- The theory of optimal taxation tries to provide consistent and rigorous tool of evaluation

Introduction to tax incidence

Atkinson and Stiglitz

“In principle, the analysis of the incidence of a tax is a straightforward matter.

We calculate the general equilibrium of the economy before the change in taxation or expenditure, and we recalculate the equilibrium afterwards; the *changes* provide a description of the incidence of the tax.

We can say whose income has gone up or down and by how much, and what prices have changed.

In almost all situations, the real incomes of individuals other than those upon whom the tax is levied will change.

Quite often the change in real income of those upon whom the tax is levied is smaller than the magnitude of the tax. We say then that the tax has been *shifted* to others in the economy.”

Who bears the burden of a tax?

Economic impacts

- Consumption patterns
 - Distribution
 - Supply of factors
- > Growth, employment, industrial structure
- Allocative efficiency
- Distributive equity

Concepts

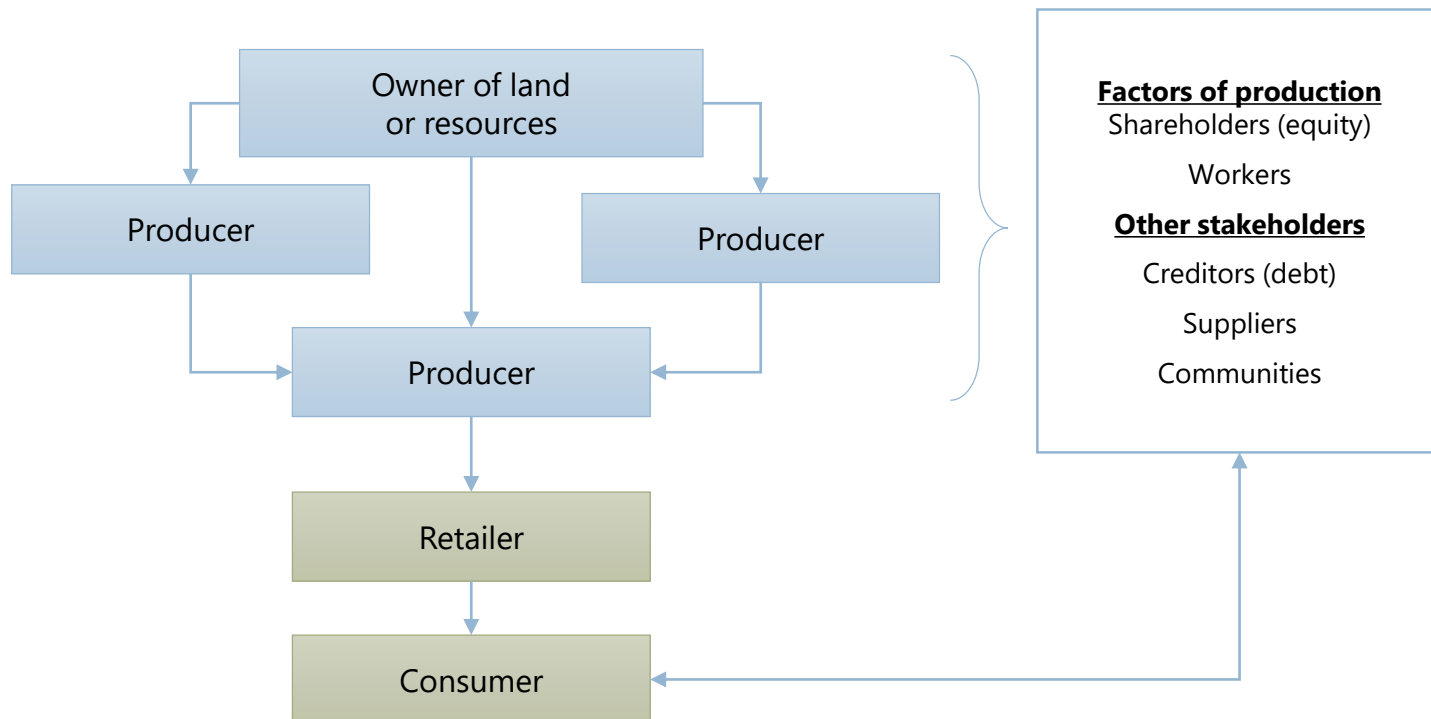
- Statutory and economic incidence
- Shifting forwards and backwards
- Equivalent taxes
- Equivalence of taxes and subsidies

Analytical tools

- Partial equilibrium
- General equilibrium

Economic factors that influence incidence

- Elasticities
- Degree of competition



- Tax incidence is the study of the effects of tax policies on prices and the distribution of utilities
- Ideally, we would characterize the effect of a tax change on utility levels of all agents in the economy
- In practice: aggregate economic agents into a few groups
 - Producer vs. consumer (tax on cigarettes)
 - Source of income (labour vs. capital)
 - Income level (rich vs. poor)
 - Region or country (local property taxes)
 - Across generations (social security reform)
- In general tax incidence analysis is informative about signs and comparative statics but is inconclusive about magnitudes

- Consider the following argument: Government should tax capital income because it is concentrated at the high end of the income distribution
- Neglects general equilibrium price effects
 - Tax might be shifted onto workers
 - If capital taxes result in less savings and/or capital flight then over time the capital stock may decline, driving return to capital up and wages down
 - Some argue that capital taxes are paid by workers and therefore increase income inequality

- Assumes
 - Two good economy
 - Only one relative price so partial and general equilibrium are same
 - Might be realistic if:
 - the market being taxed is small
 - there are no close substitutes/complements (demands is independent)
 - We can think of good one being the taxed good, and good two being all other commodities.
 - Tax revenue is “thrown into the ocean”
 - Perfect competition among producers
- Commodity taxation - Government levies an excise tax on good x
 - Excise or specific tax
 - Ad-valorem tax: fraction of prices (e.g. VAT)

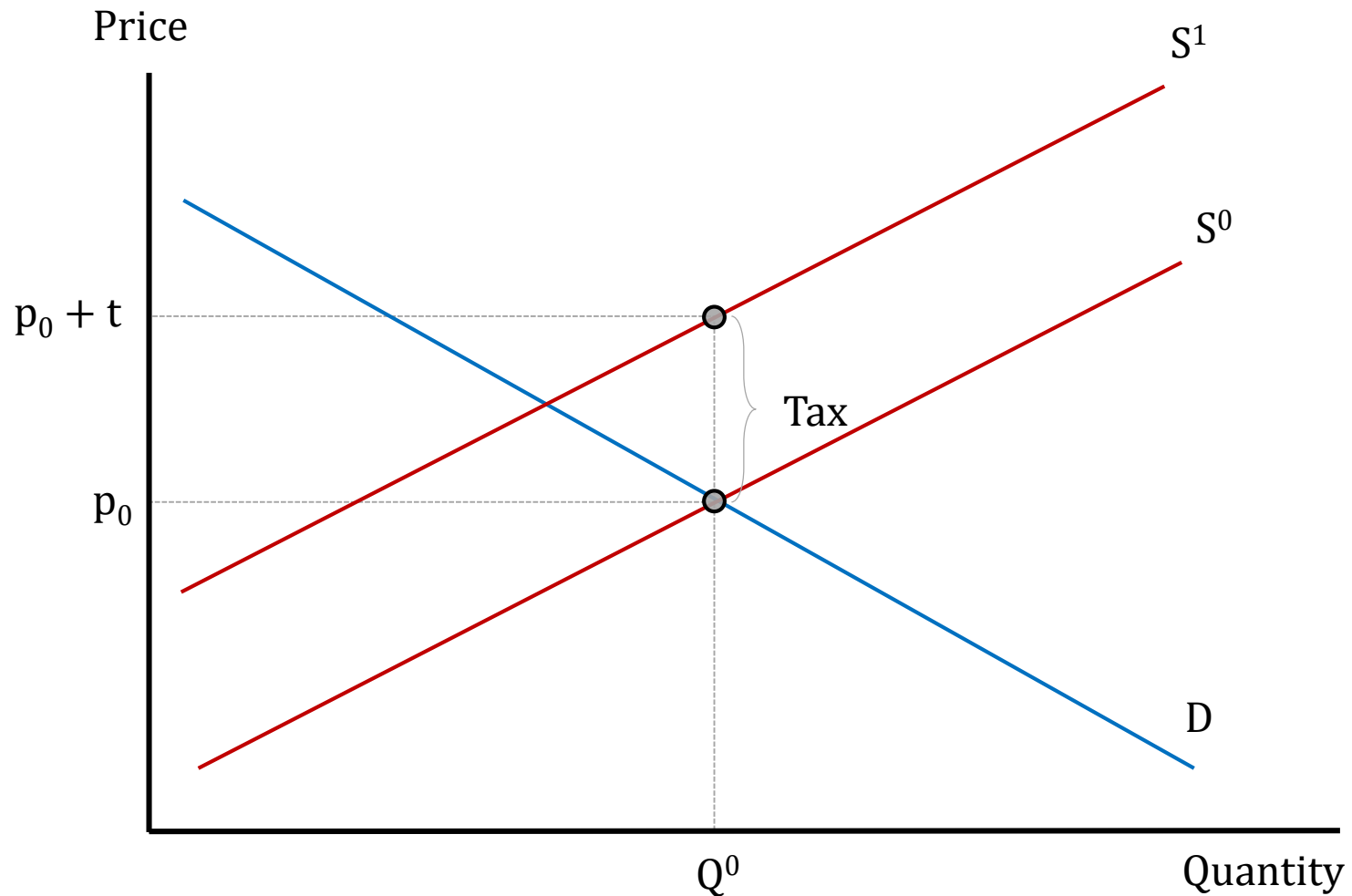
Table 4.6 Changes in specific excise duties, 2019/20

Product	Current excise	Proposed excise	Percentage change	
	duty rate	duty rate	Nominal	Real
Malt beer	R95.03 / litre of absolute alcohol (161,56c / average 340ml can)	R102.07/ litre of absolute alcohol (173,51c / average 340ml can)	7.4	2.2
Traditional African beer	7,82c / litre	7,82c / litre	–	-5.2
Traditional African beer powder	34,70c / kg	34,70c / kg	–	-5.2
Unfortified wine	R3.91 / litre	R4.20 / litre	7.4	2.2
Fortified wine	R6.54 / litre	R7.03 / litre	7.4	2.2
Sparkling wine	R12.43 / litre	R13.55 / litre	9.0	3.8
Ciders and alcoholic fruit beverages	R95.03 / litre of absolute alcohol (161,56c / average 340ml can)	R102.07/ litre of absolute alcohol (173,51c / average 340ml can)	7.4	2.2
Spirits	R190.08 / litre of absolute alcohol (R61.30 / 750ml bottle)	R204.15 / litre of absolute alcohol (R65.84 / 750ml bottle)	7.4	2.2
Cigarettes	R15.52 / 20 cigarettes	R16.66 / 20 cigarettes	7.4	2.2
Cigarette tobacco	R17.44 / 50g	R18.73 / 50g	7.4	2.2
Pipe tobacco	R4.94 / 25g	R5.39 / 25g	9.0	3.8
Cigars	R82.31 / 23g	R89.72 / 23g	9.0	3.8

Source: National Treasury

Specific tax in a competitive market

- A specific tax on a firm's output
- Effective cost of production is increased by the amount of the tax
- In aggregate, the supply curve shifts up by the amount of the tax



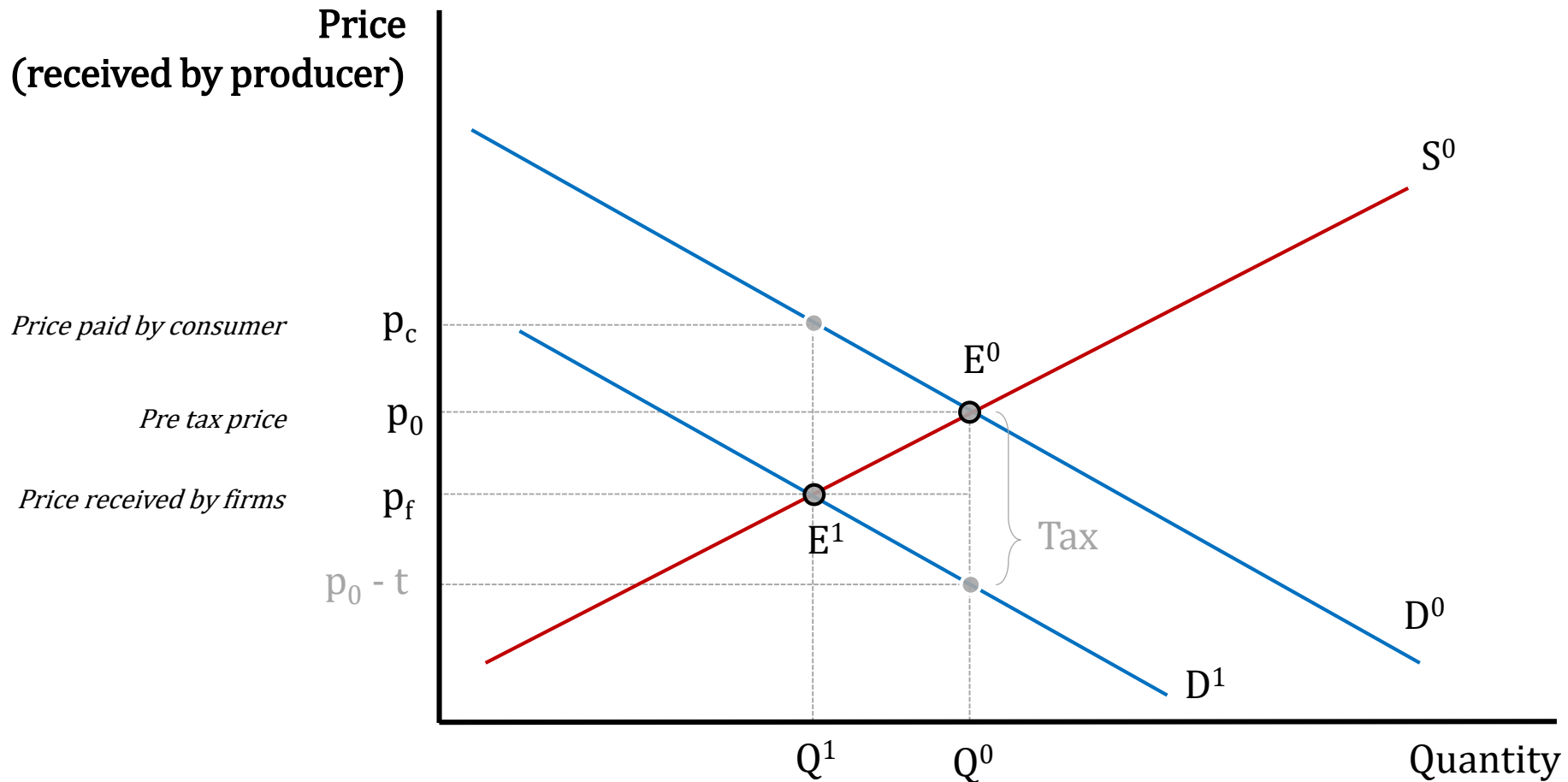
Specific tax in a competitive market

- The burden of the tax is shared between consumers and firms
- **What happens if the tax was paid by the consumer (or the retailer at the point of sale)?**

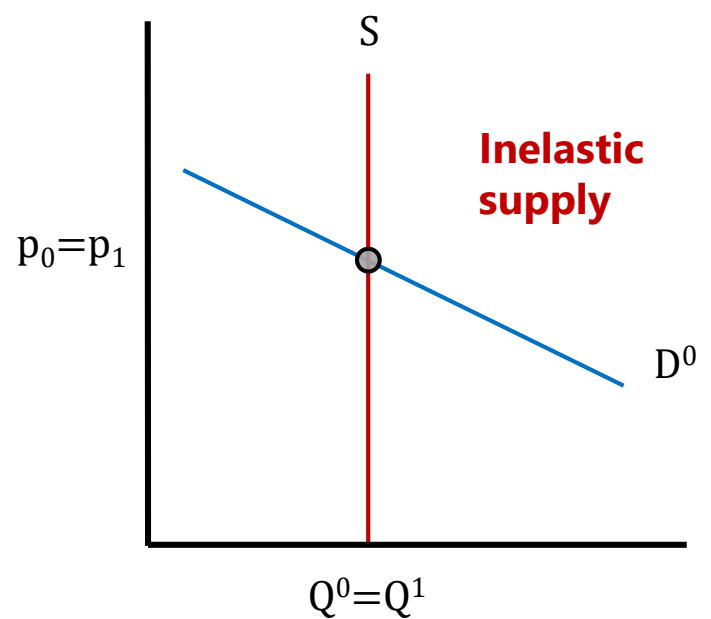
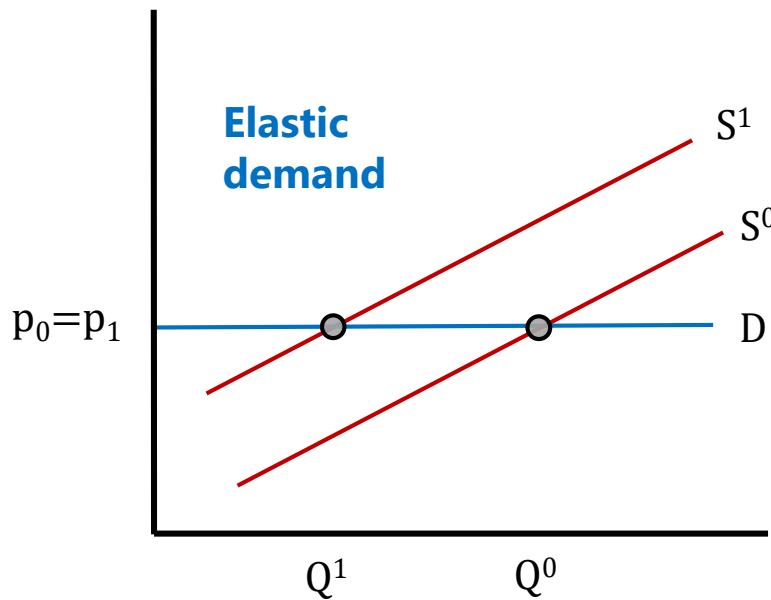
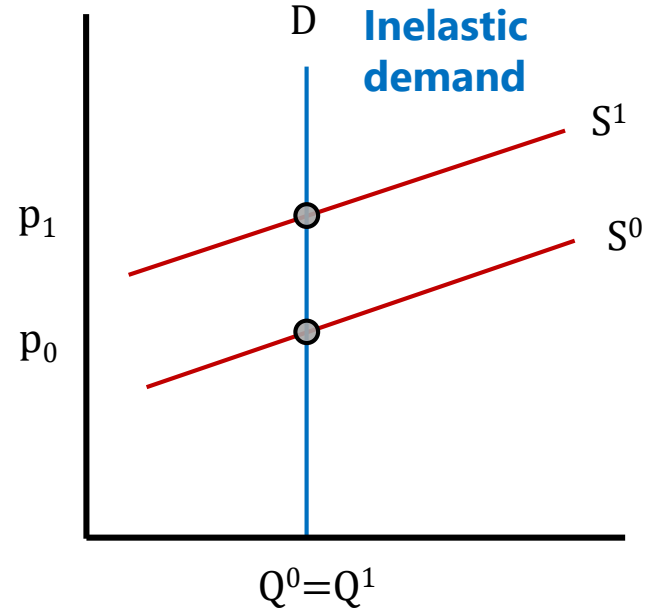
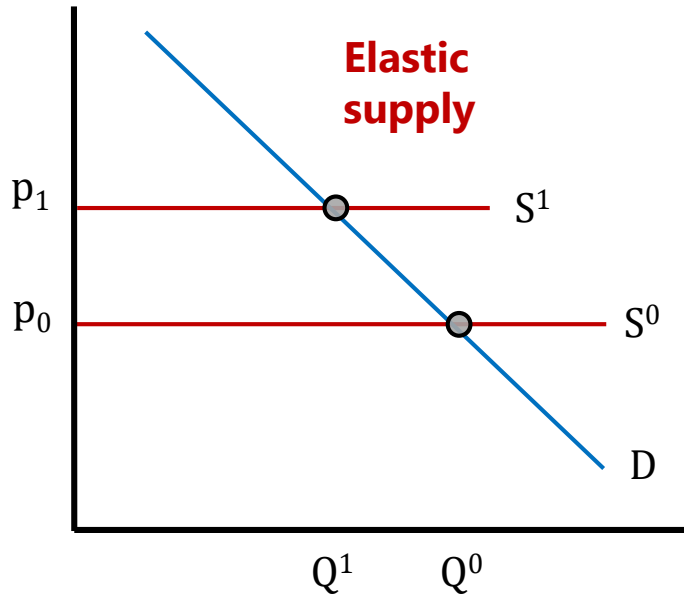


Specific tax in a competitive market

- What happens if the tax was paid by the consumer (or the retailer at the point of sale)
- The statutory burden is different, but the economic burden is exactly the same.



Elasticity: limiting cases



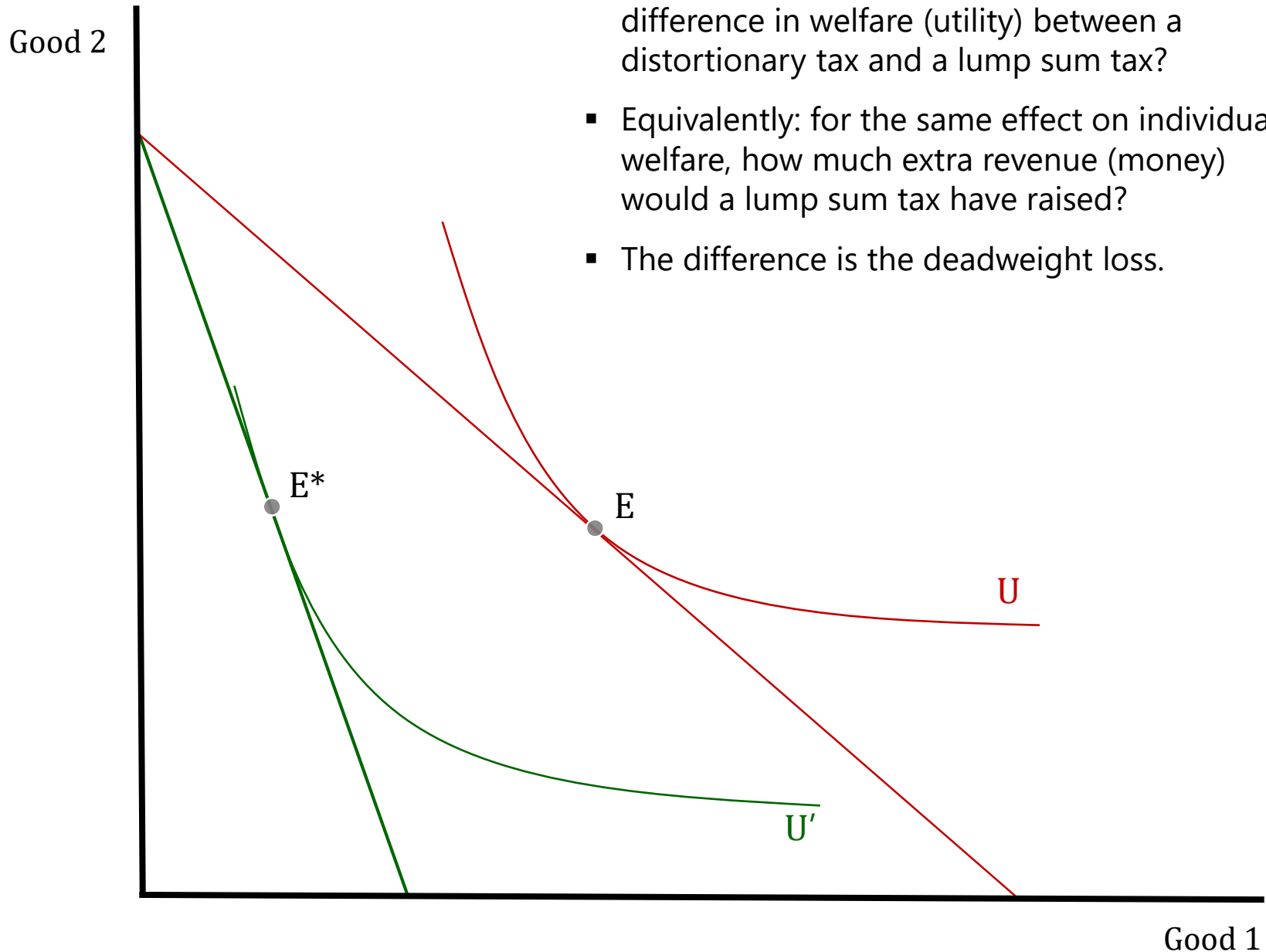
- Economic incidence describes who actually bears the burden – whose real income or welfare changes as a result of the tax
- It does not depend on who pays over the money (statutory incidence)
- In competitive markets, incidence depends on the elasticity of demand and supply
 - If demand is perfectly elastic, consumers bear no burden
 - If supply is perfectly elastic, producers bear no burden.
 - If demand is perfectly inelastic, consumers bear the full burden
 - If supply is perfectly inelastic, producers bear the full burden

- The same principles apply to factor markets
 - The incidence of a tax on a factor in a competitive market depends on the elasticity of supply and demand for the factor.
 - Factors with inelastic supply bear the burden of the tax
 - Elastic factors can shift the burden.
 - Mobility of capital and labour?
 - More on this next week.
- Land tax
 - The supply of (unimproved) land is fixed
 - Therefore the total burden of the tax will fall on landowners
 - Distinguishing land value from improvements
 - Consider a tax on oil or platinum

Introduction to taxation and efficiency

Welfare analysis of distortionary taxation

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- For the same amount of revenue what is the difference in welfare (utility) between a distortionary tax and a lump sum tax?
- Equivalently: for the same effect on individual welfare, how much extra revenue (money) would a lump sum tax have raised?
- The difference is the deadweight loss.

▪ **Income effect**

- All taxes take income away from individuals, it makes them worse off.
- Individuals make different decisions when their income changes.
- Because they are poorer they postpone their retirement, they cannot enjoy much leisure, spouses take on work instead of looking after children...

▪ **Substitution effect** Efforts to avoid taxes by substituting non-taxed for taxed activities

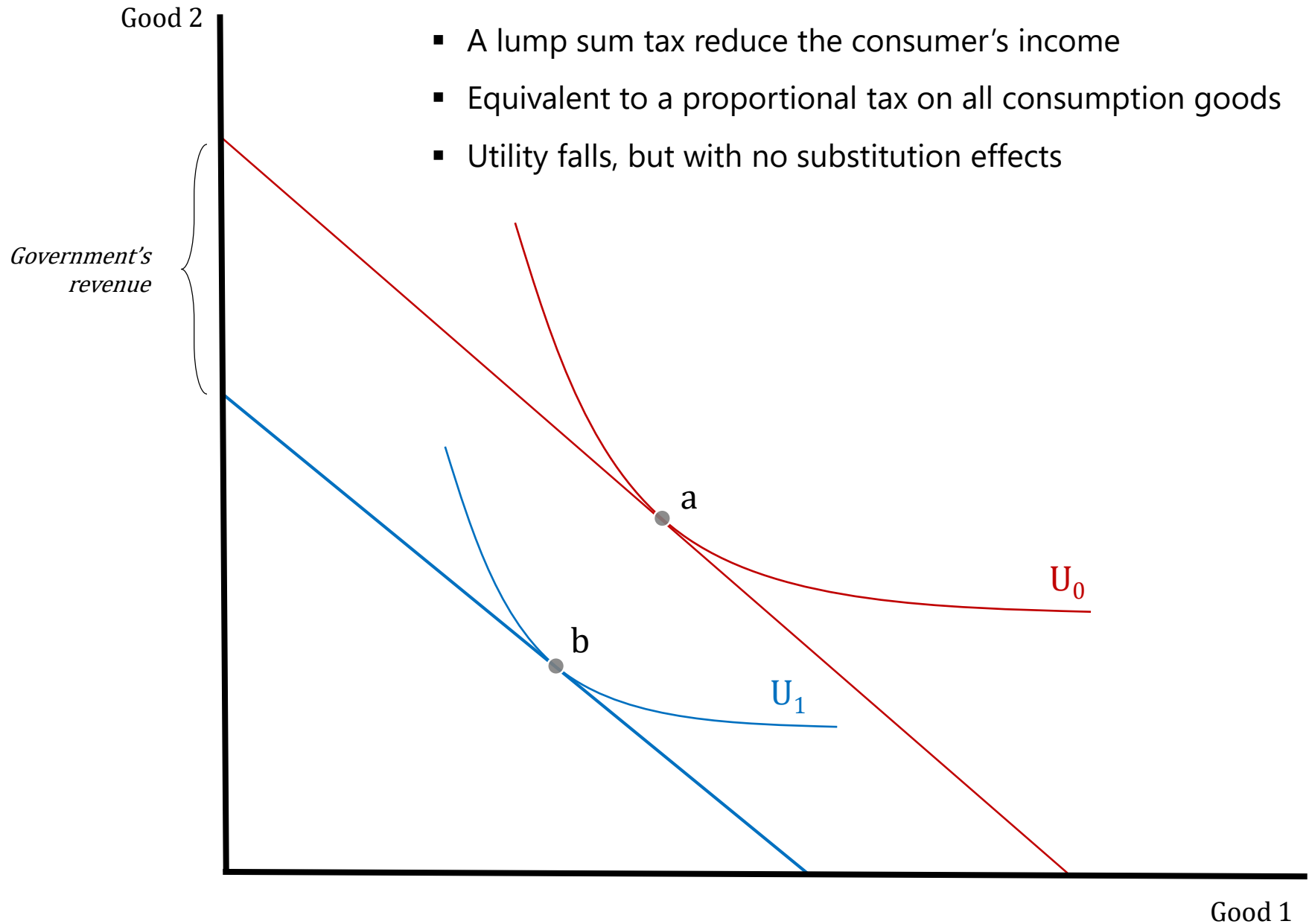
- Not all activities are taxed, or taxed at the same rate.
- Taxation changes relative prices from their economic equilibrium.
- This diverts economic activity from taxed to untaxed areas, or from areas of higher taxes to areas of lower tax.
- Leisure, household production, subsistence agriculture do not attract taxation. People seek out untaxed areas – for instance more leisure may mean putting in less effort on the job, retiring earlier, working shorter hours, or beginning work later in life.
- (Note the contrary impacts on work effort and retirement choice)

▪ **Financial effects**

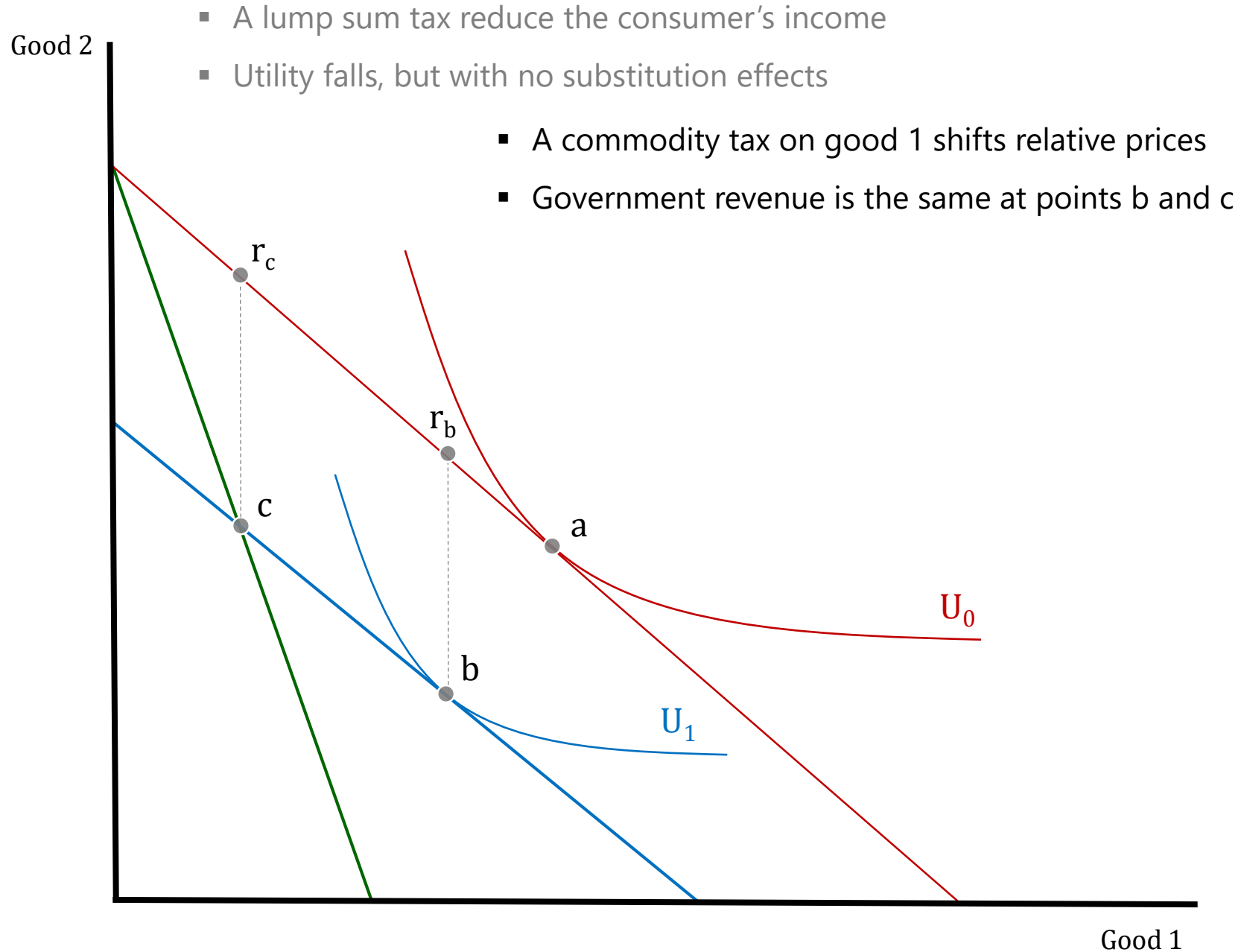
- Real activity can be mediated through different forms of payments
- People may prefer payment in cash or in kind to avoid taxation.
- Remuneration may shift from income to capital
- Finance may be provided in the form of debt rate then equity.

Income and substitution effects

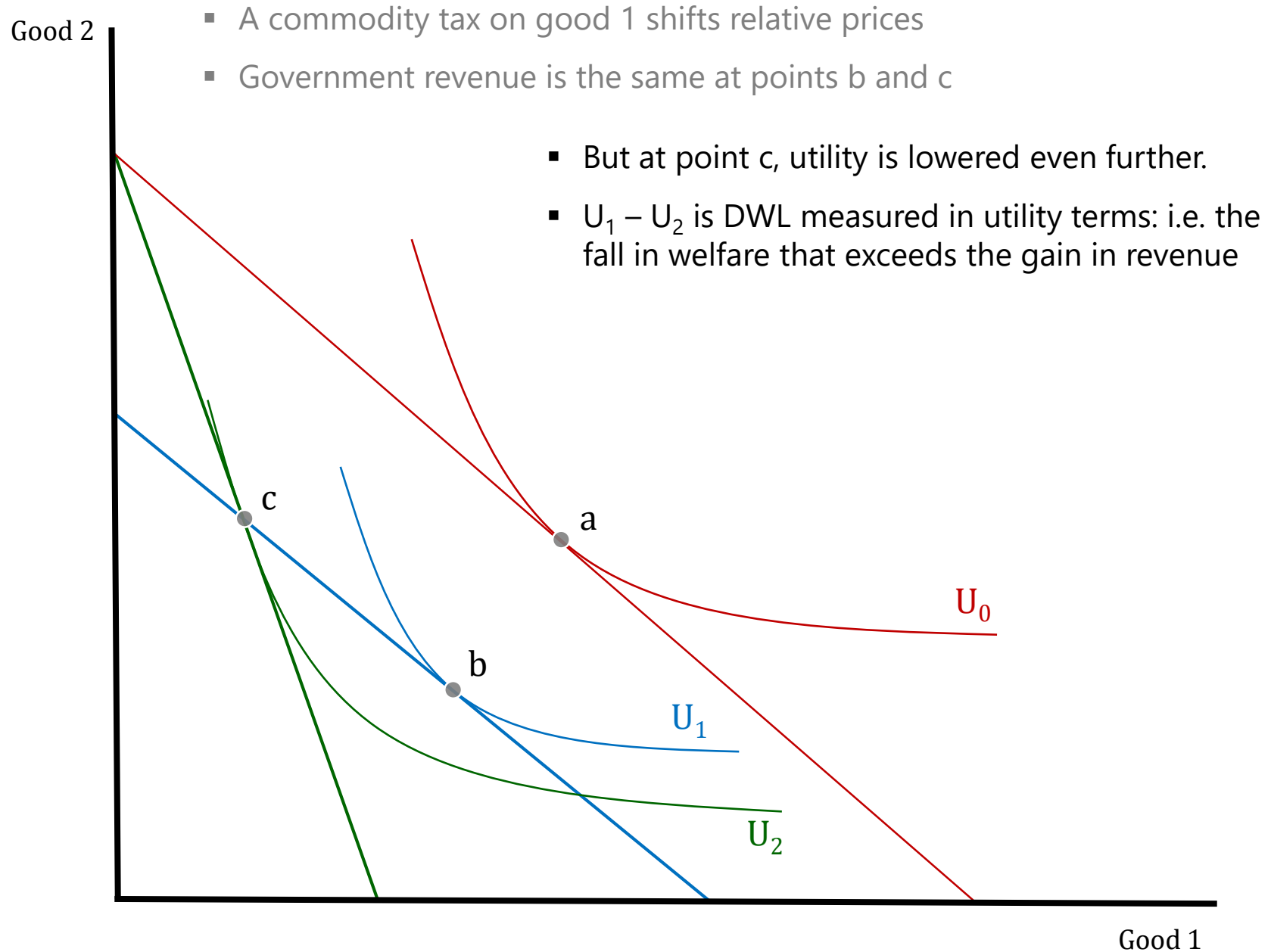
- A lump sum tax reduce the consumer's income
- Equivalent to a proportional tax on all consumption goods
- Utility falls, but with no substitution effects



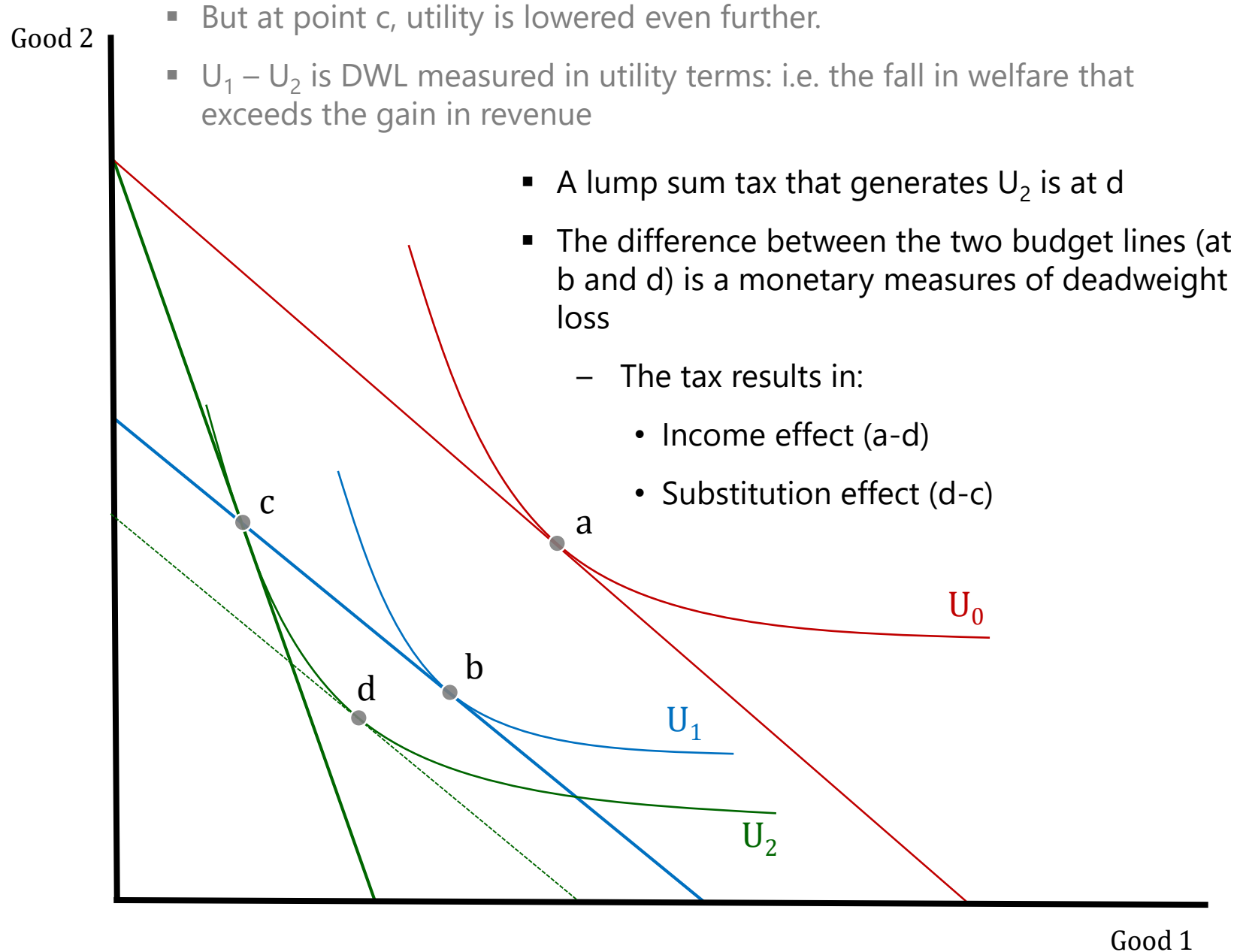
Income and substitution effects



Income and substitution effects

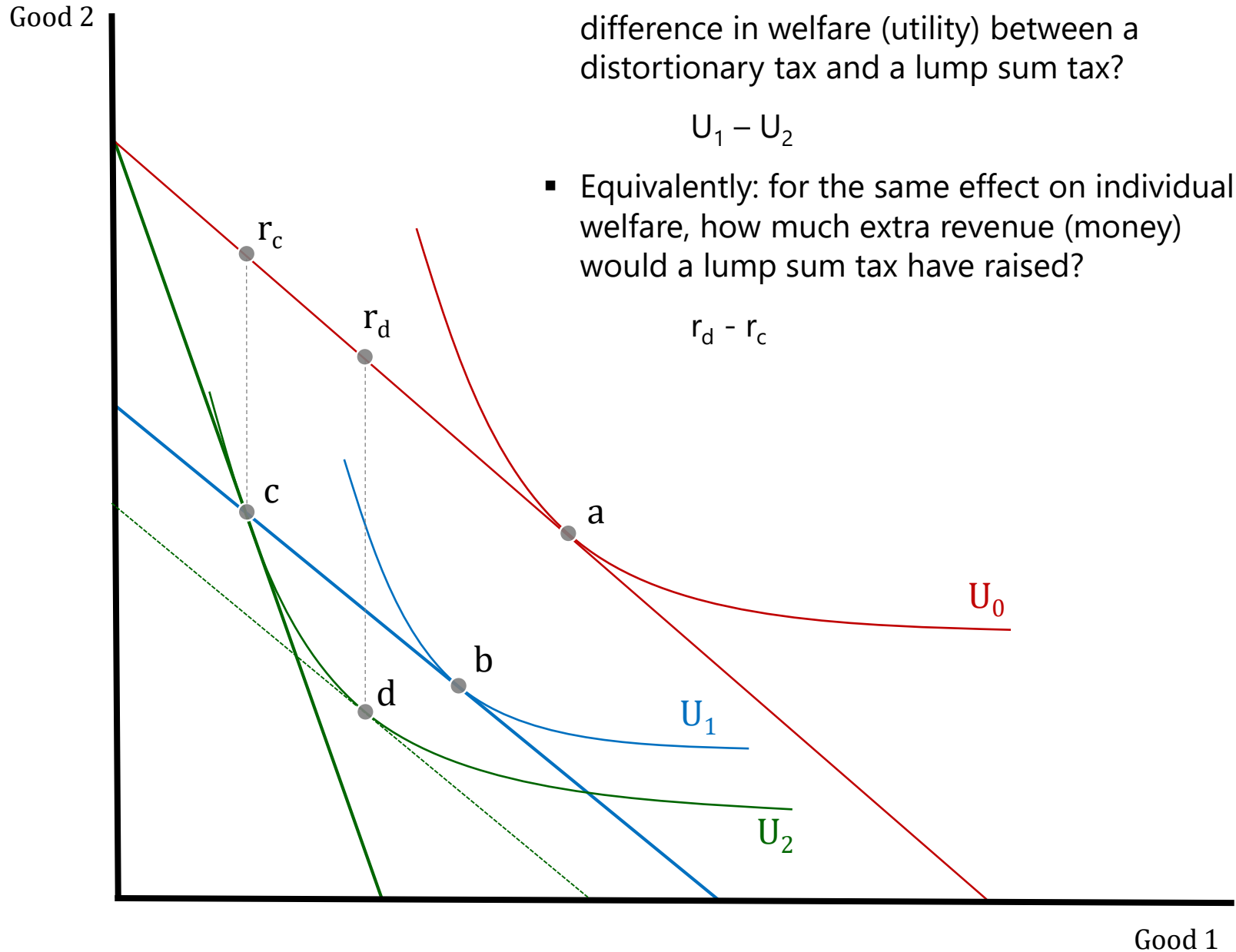


Income and substitution effects

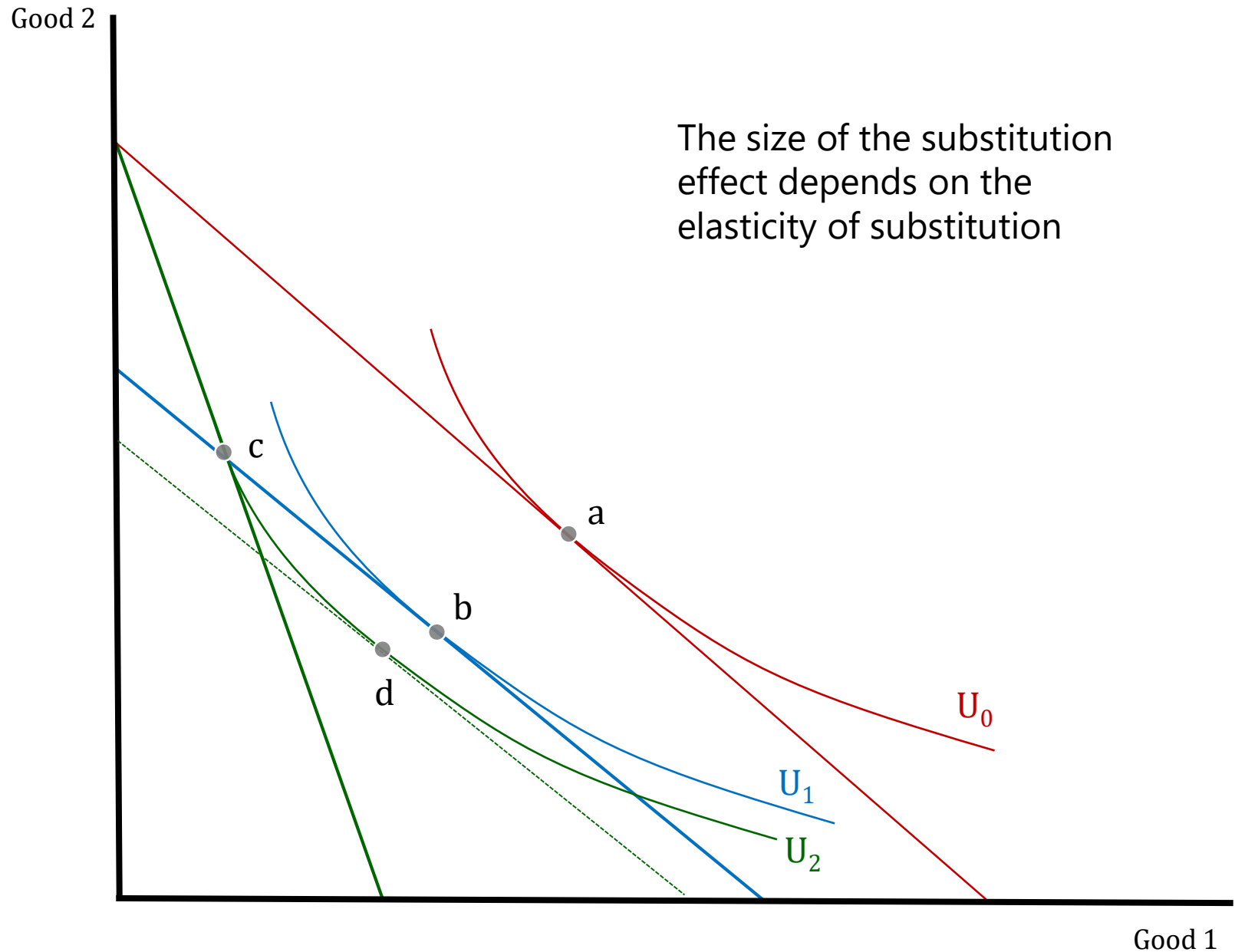


Magnitudes of the income and substitution effect

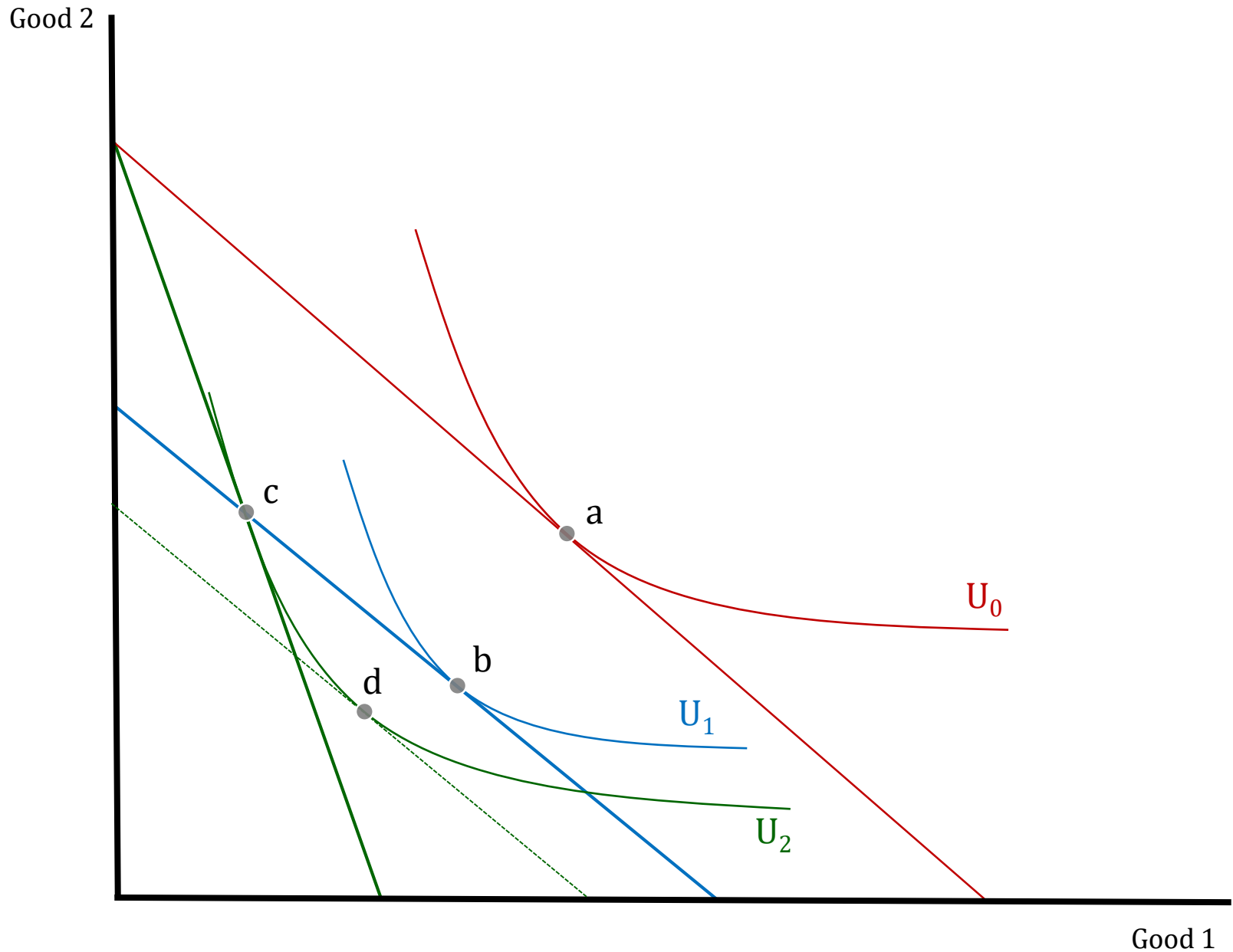
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Magnitudes of the income and substitution effect

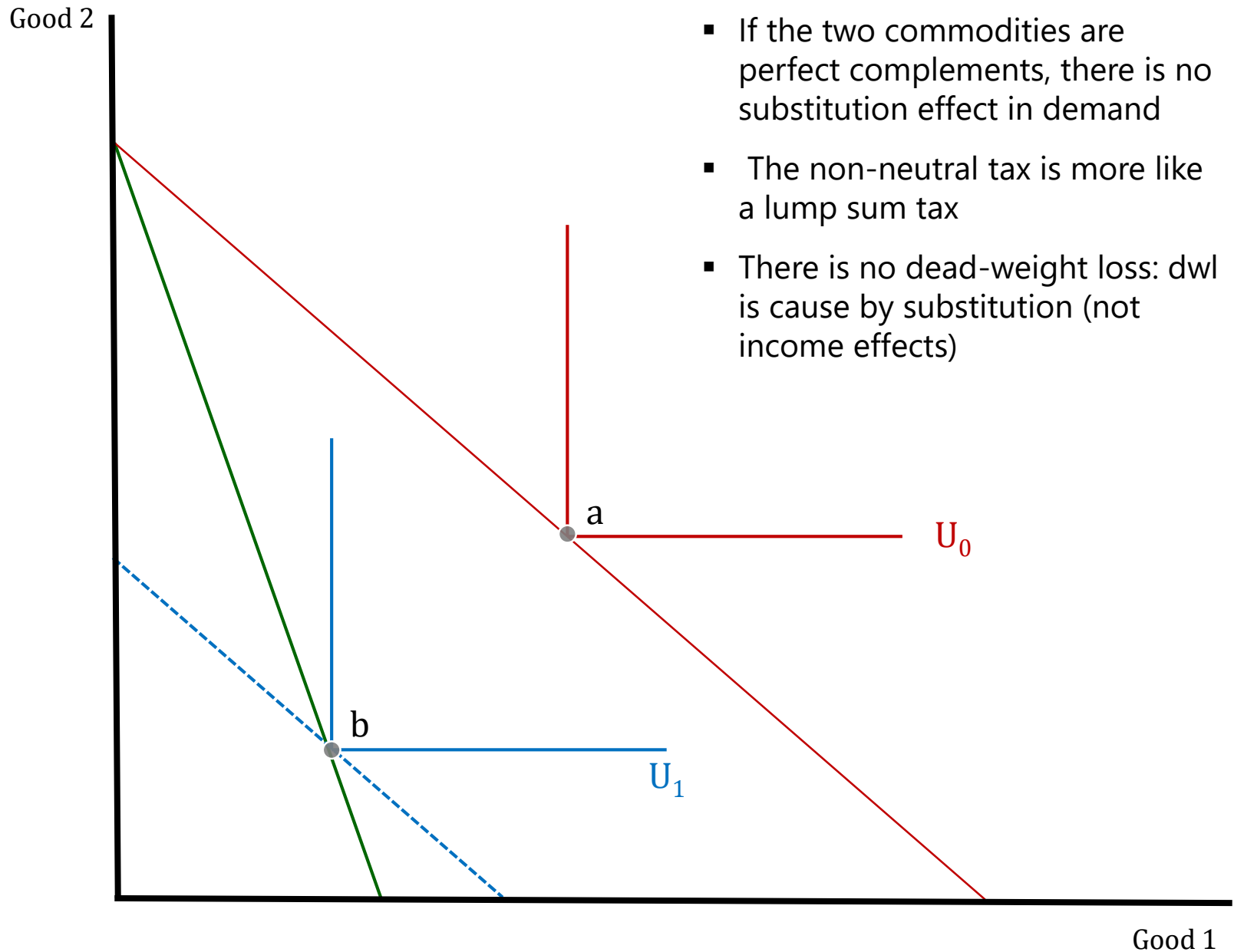


Magnitudes of the income and substitution effect

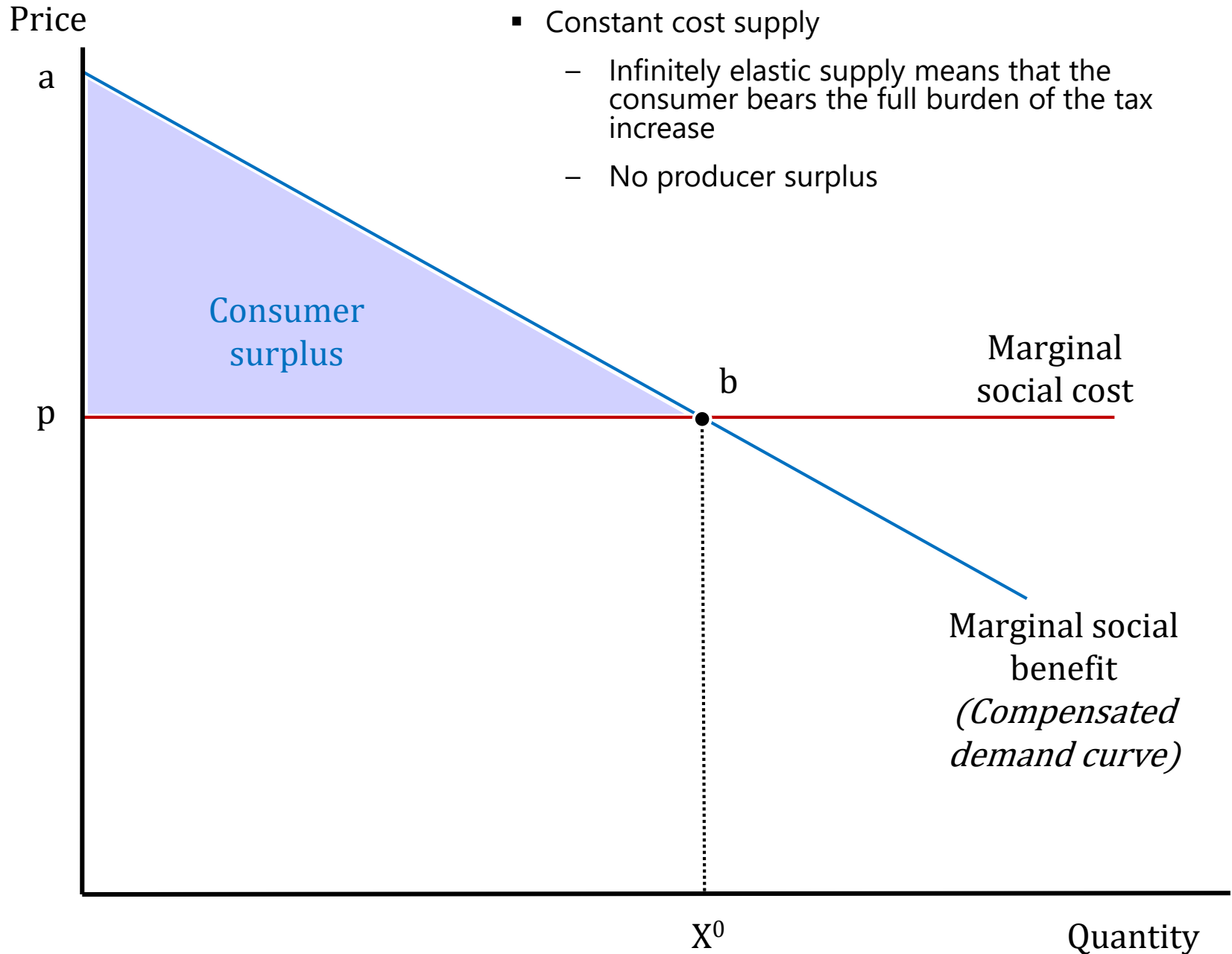


Magnitudes of the income and substitution effect

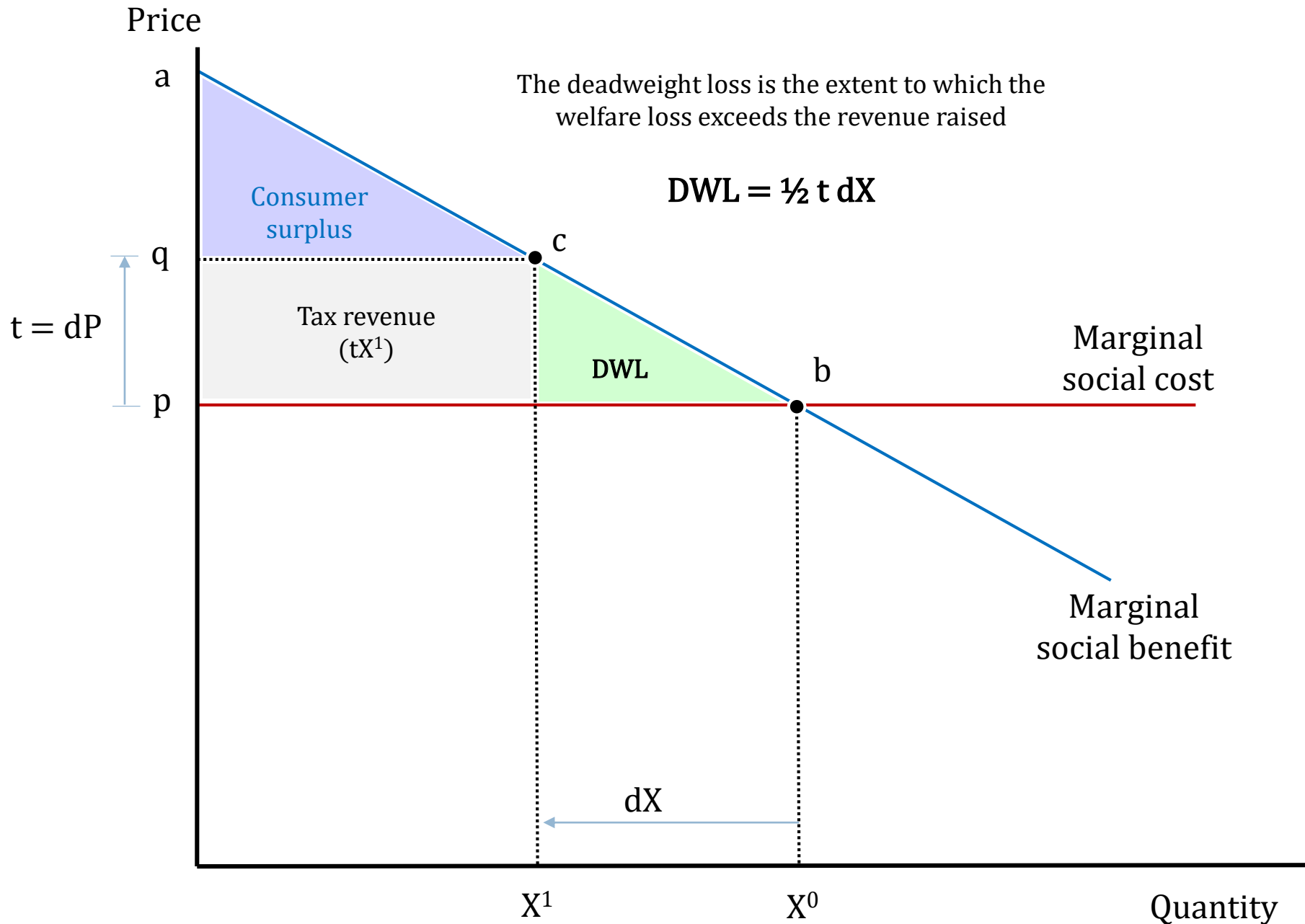
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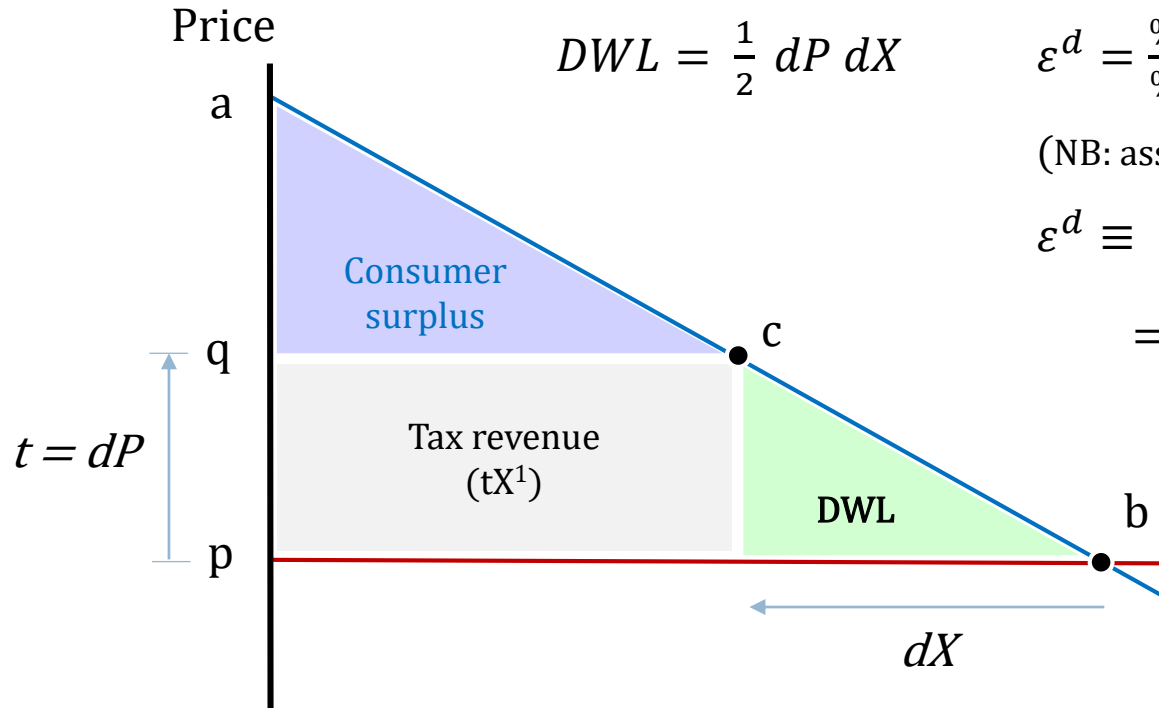
Deadweight loss (excess burden)



Deadweight loss (excess burden)



Deadweight loss (excess burden)



$$\varepsilon^d = \frac{\% \Delta Q}{\% \Delta P} = \frac{dQ/Q}{dP/P}$$

(NB: assume constant elasticity in log space)

$$\varepsilon^d \equiv \frac{P dX}{X dP}$$

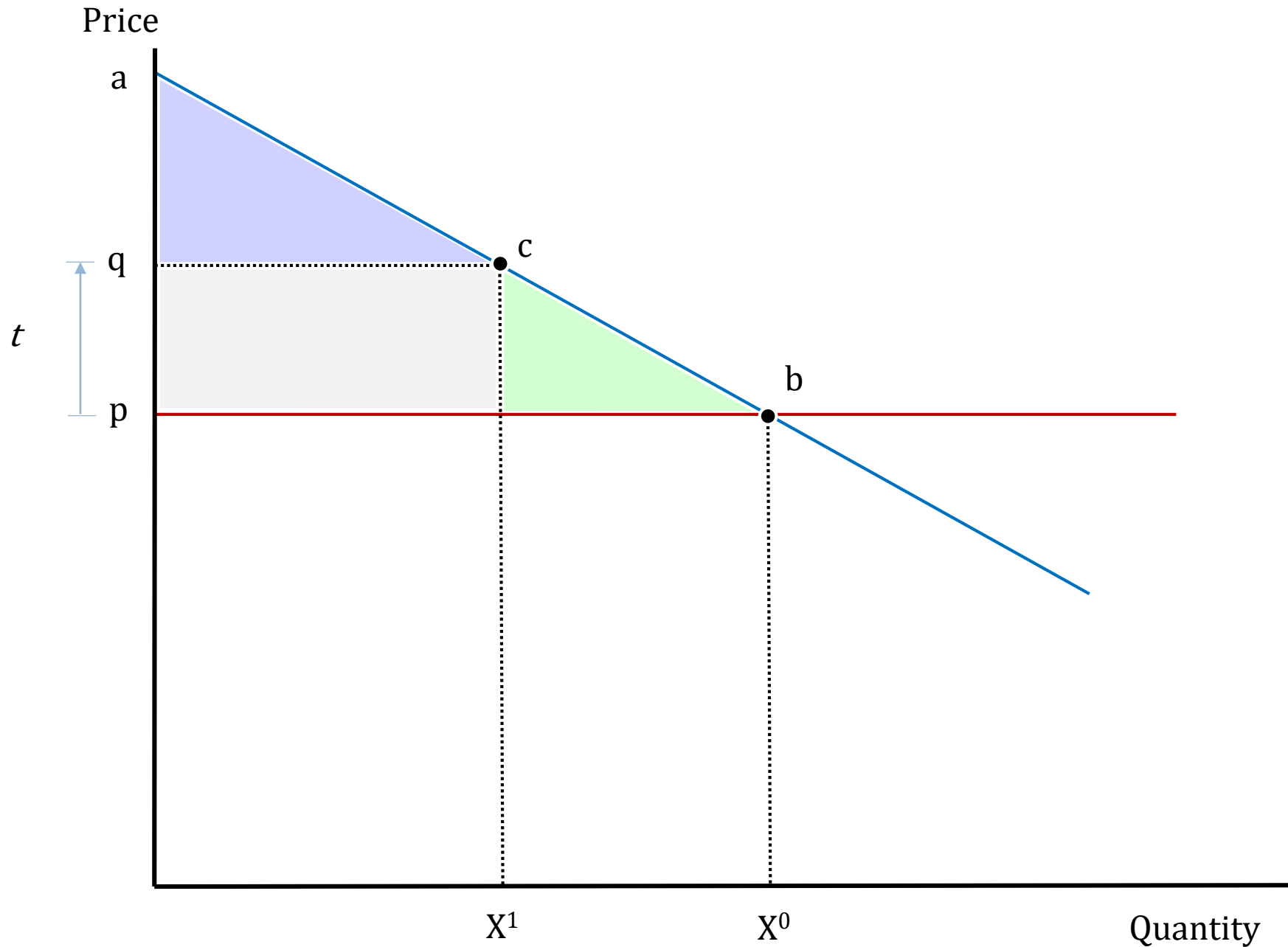
$$\Rightarrow dX = \frac{dP}{P} \varepsilon^d X$$

$$\text{or } dX = \frac{t}{P} \varepsilon^d X$$

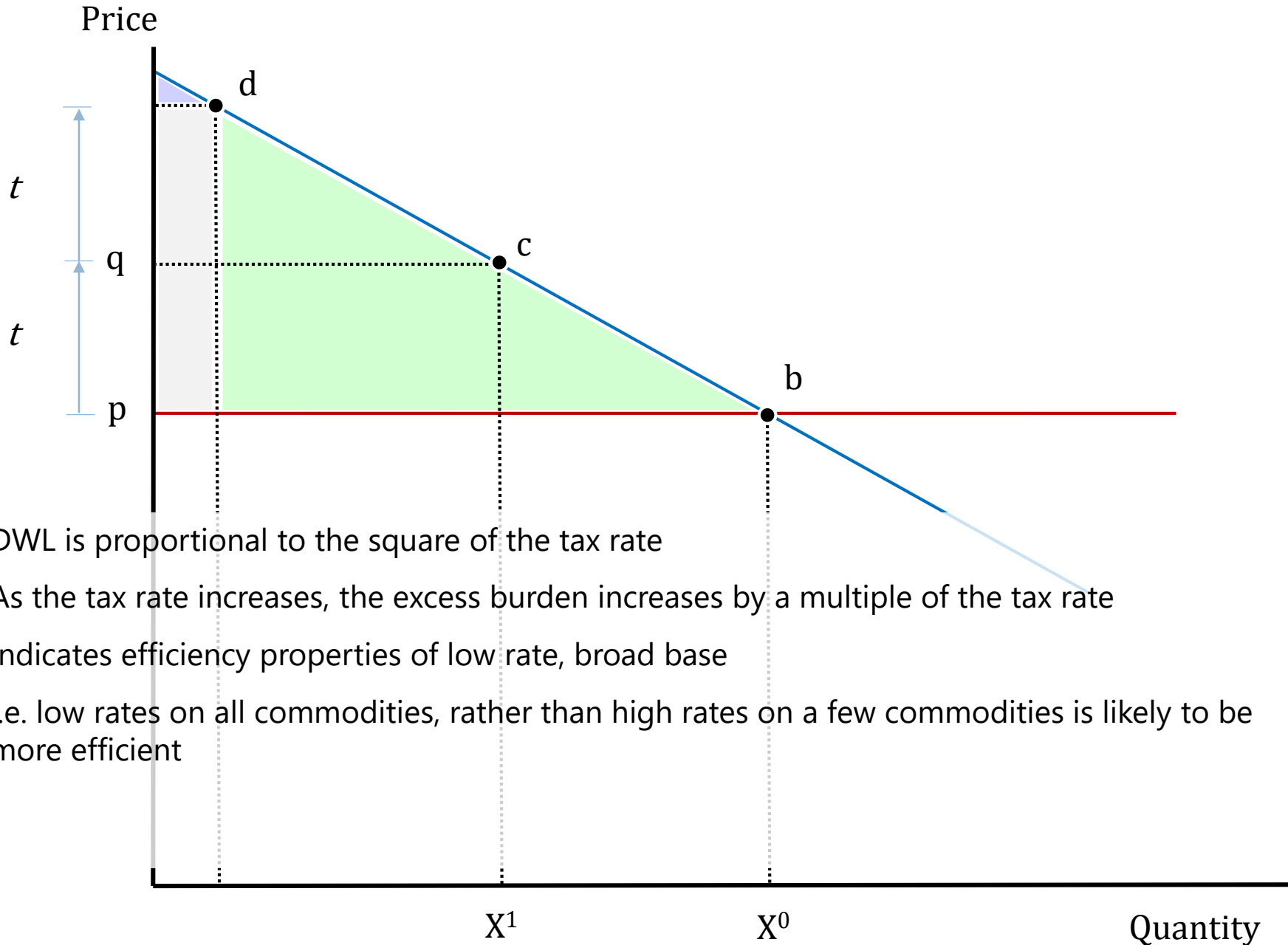
$$\Rightarrow DWL = \frac{1}{2} \left(\frac{t}{P} \right)^2 \varepsilon^d X P$$

- DWL is proportional to:
 - The square of the tax rate
 - The elasticity of demand

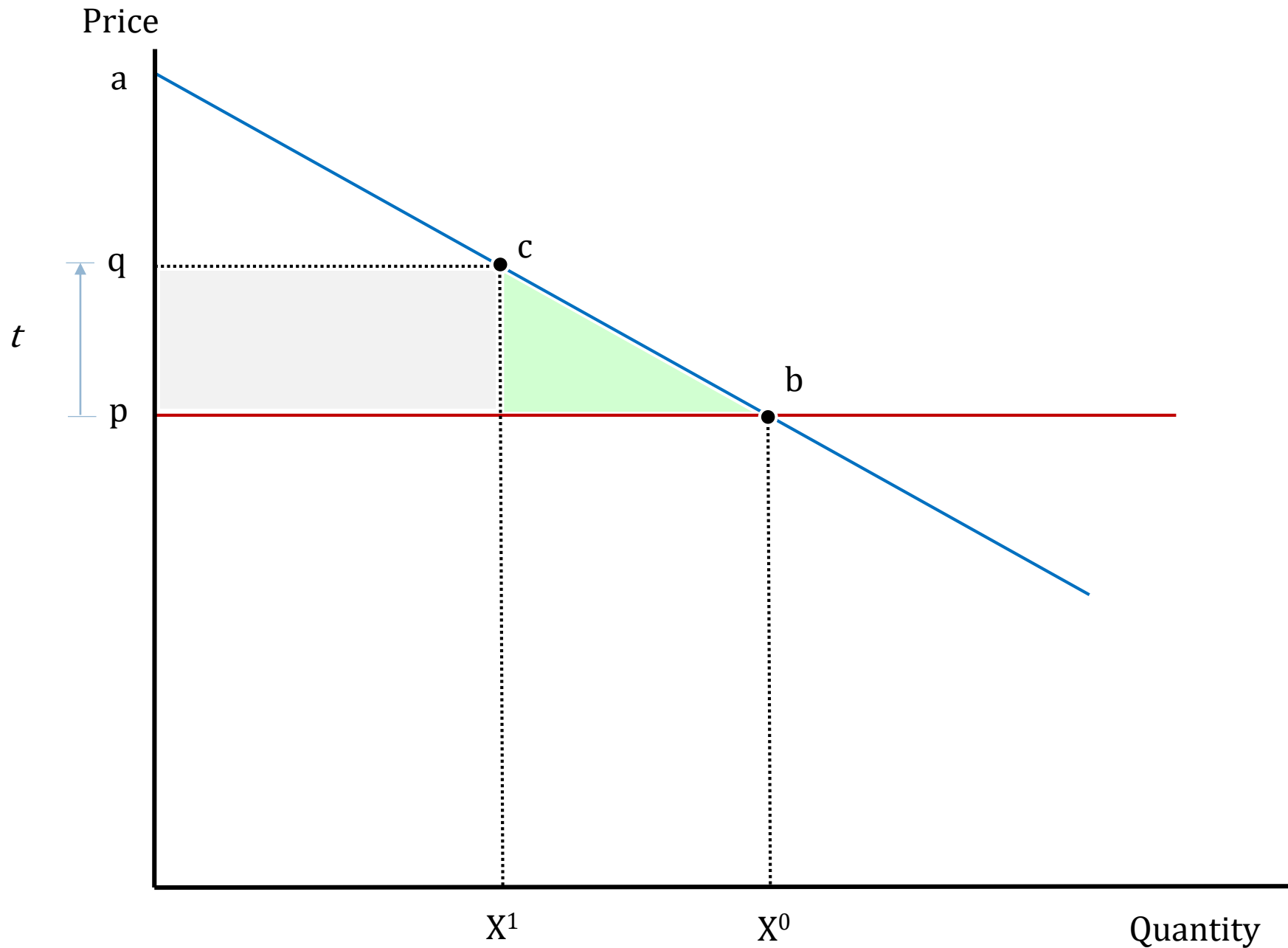
Proportional to tax rate



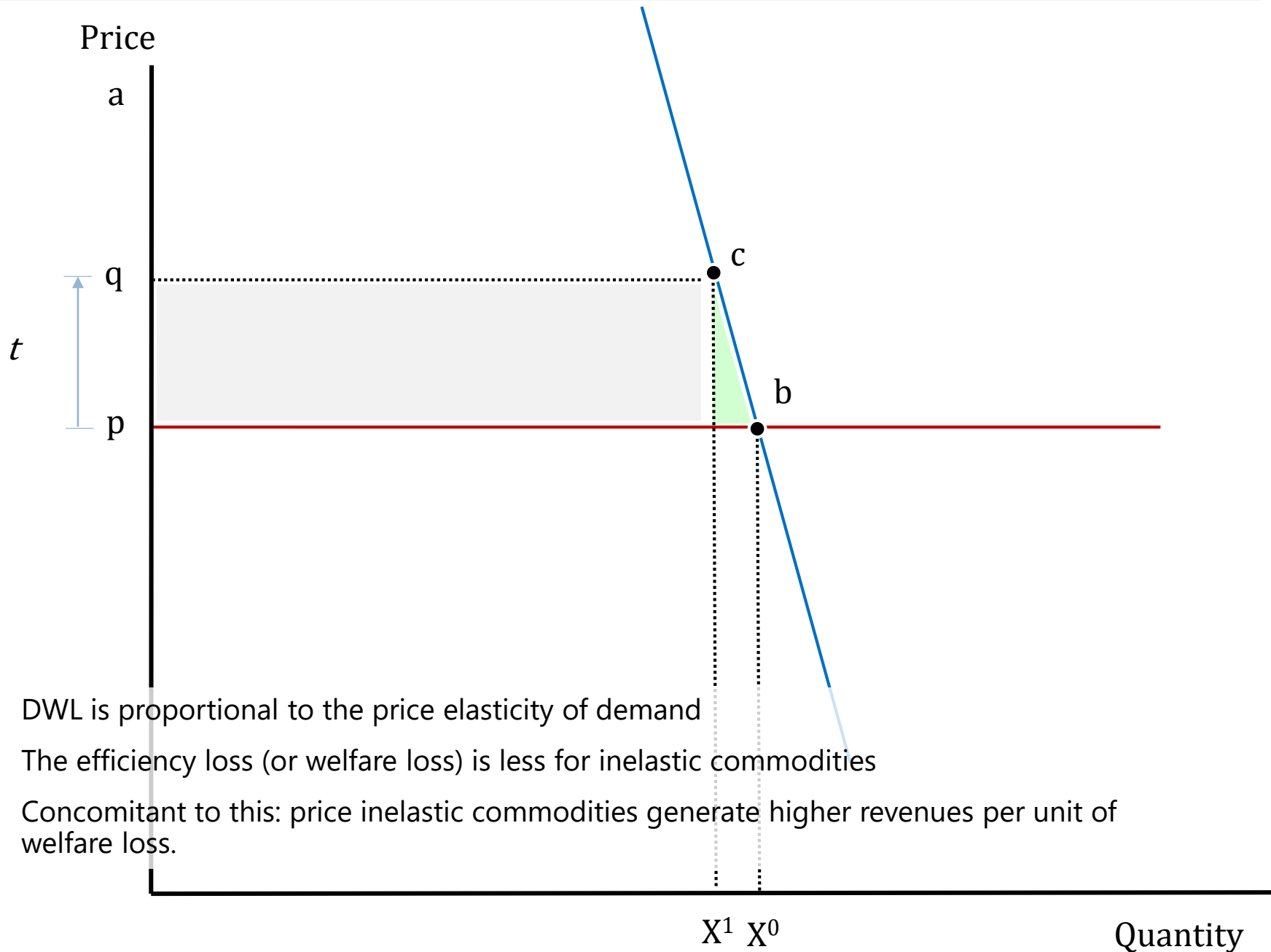
Proportional to tax rate



Proportional to elasticity



Proportional to elasticity



- DWL is proportional to the price elasticity of demand
- The efficiency loss (or welfare loss) is less for inelastic commodities
- Concomitant to this: price inelastic commodities generate higher revenues per unit of welfare loss.

- Leaving aside distributional concerns (which can be addressed on the expenditure side)
- ... focussing on how to finance a given quantum of transfers to the poor ...
- **Should we introduce two VAT rates, one for basic commodities, one for luxury goods?**

- Taxes burdens can be shifted.
- The ability to shift a tax burden depends on the elasticities of supply and demand.
- The size of the excess burden (or efficiency cost of taxation) is determined by price elasticities, the size of the tax base and the tax rate.
- The more inelastic demand the greater the revenue collection and the smaller the excess burden.
- As the tax rate increases, the excess burden (distortion and welfare impact) increases by a multiple of the tax rate
- Low tax rates on a wide base will lead to lower excess burdens and more tax revenue than high tax rates on a narrow base.
- Excess burden implies efficiency losses compared to the market benchmark. At the same time there are distributional consequences for where the tax is shifted.

Introduction to optimal taxation

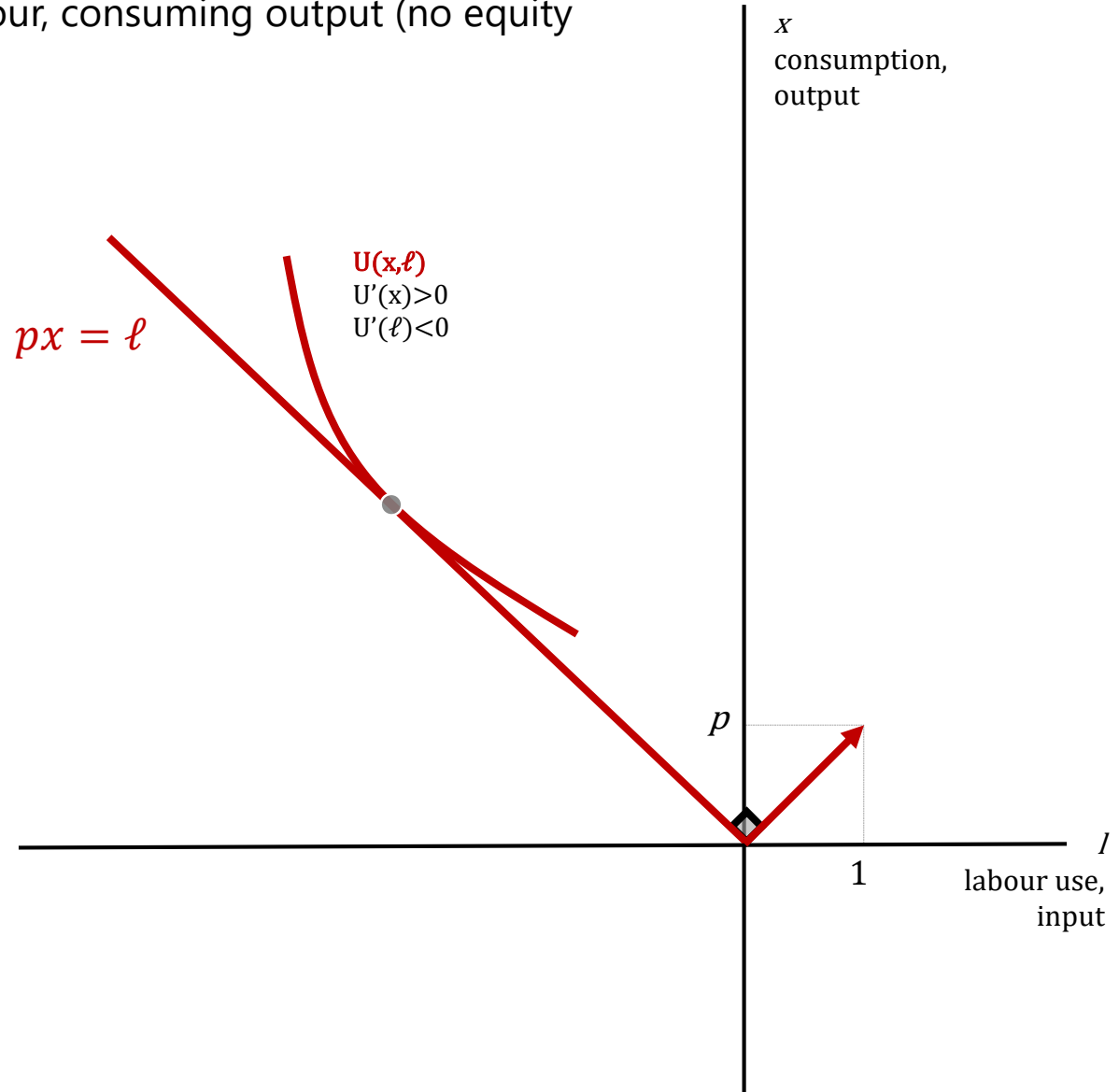
What is optimal taxation?

- Pareto efficient tax structures minimize distortions
- How do we compare or order two pareto-efficiency tax systems?
- Combine (positive) analysis about efficiency and incidence to reach (normative) conclusions about the optimal design of the whole tax system
- **The set of taxes that results in the highest level of welfare while raising the revenue required by government**
 - an objective function that takes account of efficiency and equity concerns
 - Consumers left free to choose their most preferred consumption plans at the resulting prices
 - Firms must continue to maximize profits
 - Government sets taxes on uses of income in order to accomplish two objectives: Raise a total amount of revenue (R), Minimize the utility loss for agents in the economy
 - Market clearing prices

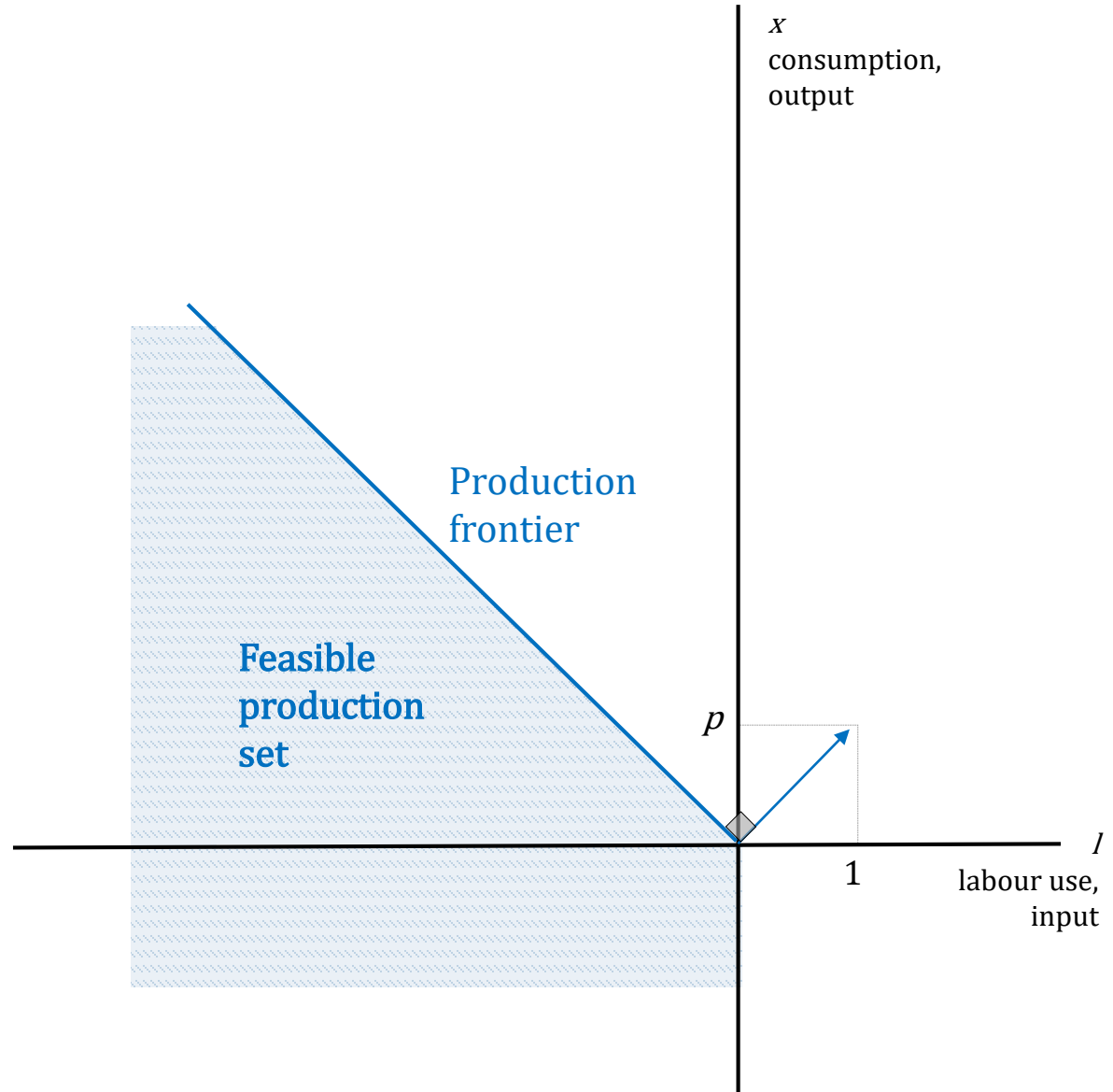
- Mainly deals with commodity taxes (which are linear)
- Rule out the possibility of lump sum taxes
- At least one good is not taxed (e.g. leisure) (so relative prices must change)
- Standard assumption : Production prices are fixed (infinitely elastic supply curve)
- For now, we assume that income tax is not an option (which is the case in many developing countries)
- The questions is: **What is the least distortionary pattern of commodity taxation?**

One consumer, two goods

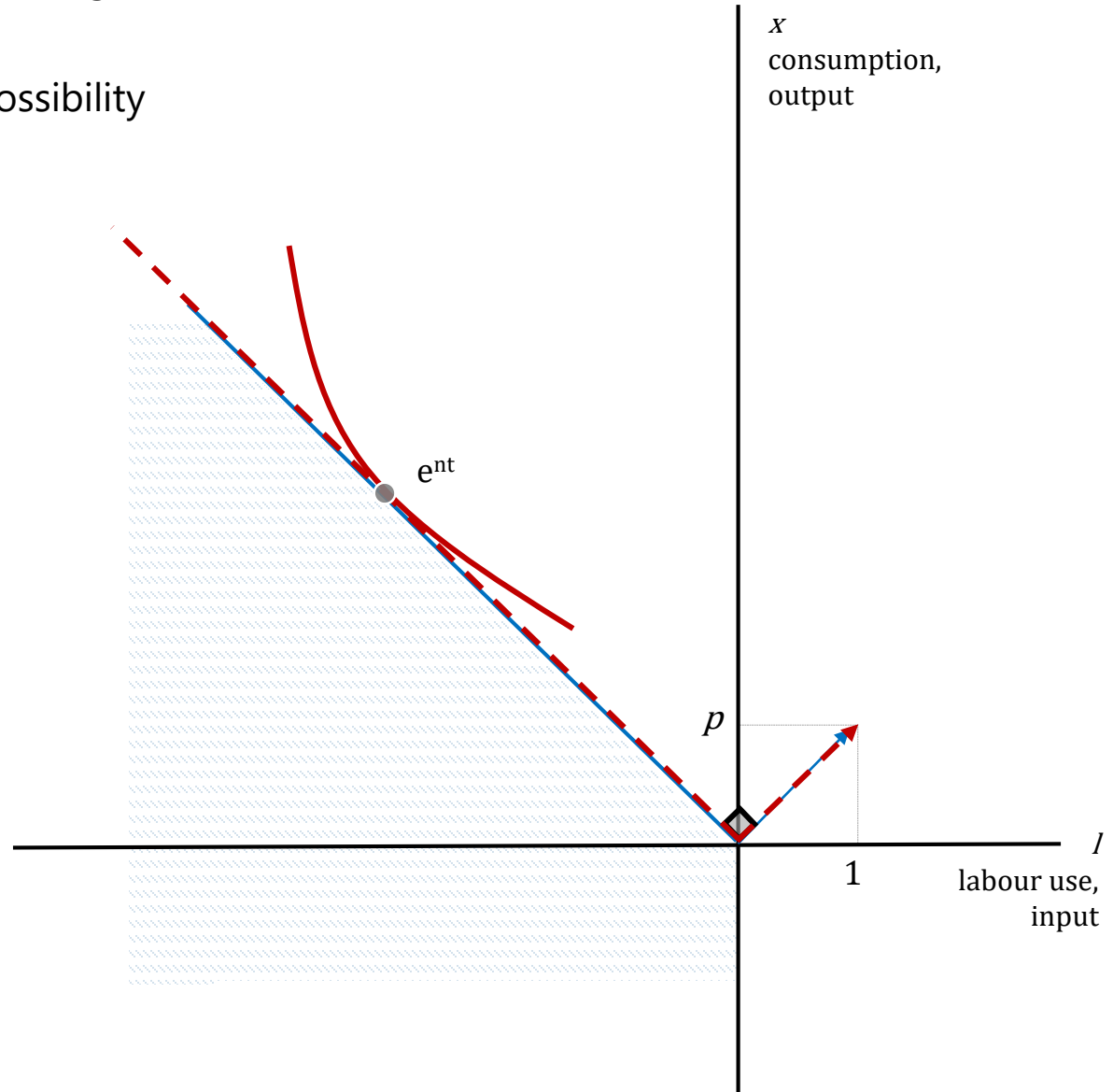
- **Two goods:** Labour (input) and consumption (output)
- **One consumer:** supplying labour, consuming output (no equity issues!)
- Labour is the numeraire and therefore wages are 1
- The price vector is orthogonal to the budget constraint



- **One firm** (demanding labour supplying output)
- The price of output (p) is set in competitive equilibrium (!) at the profit maximizing level – i.e parametric
- Production function: **constant returns to scale**
- So the firm is indifferent to points on the production frontier
- Inside the frontier there is **production inefficiency**

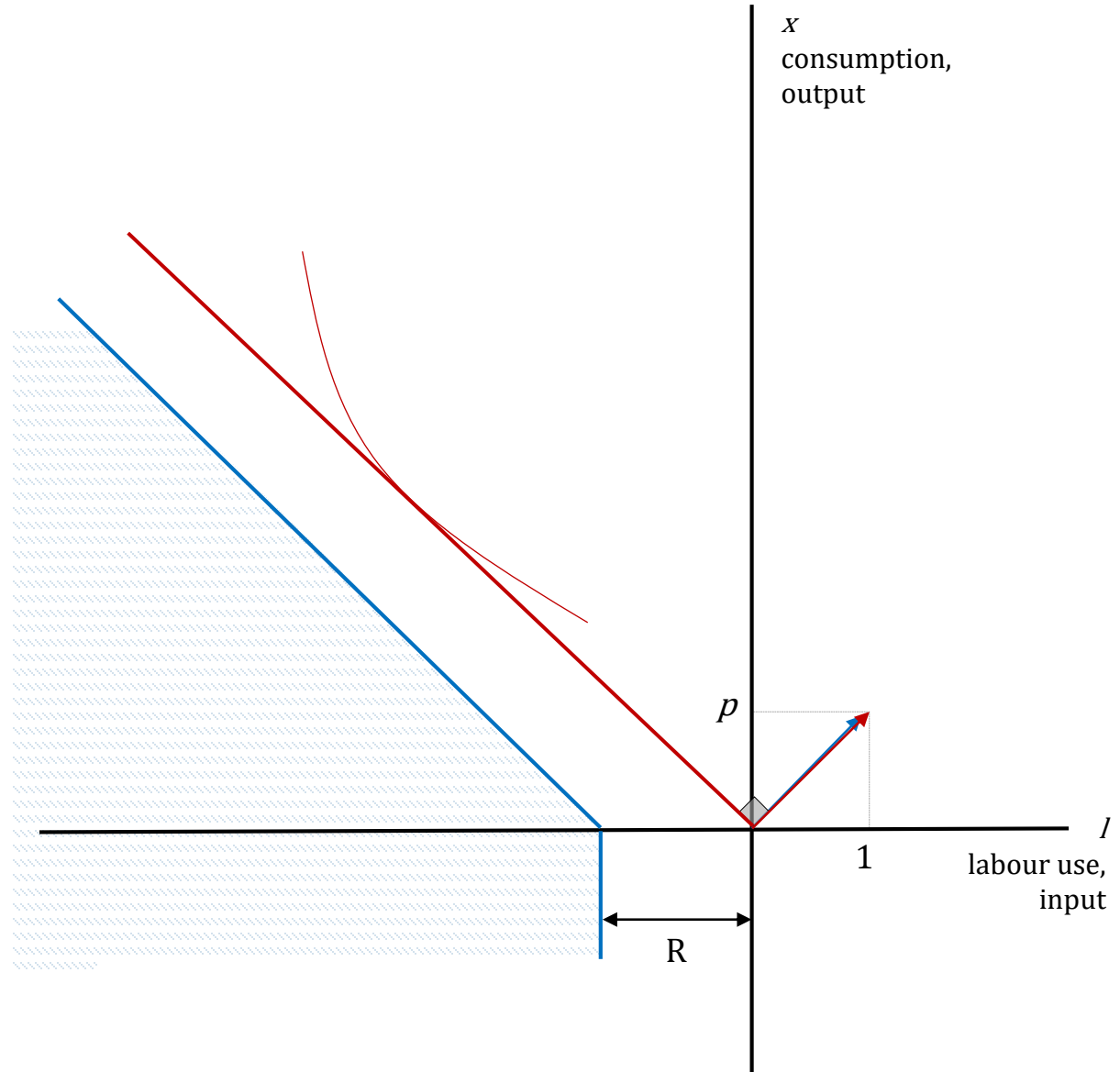


- Utility is maximized subject to budget constraint
- Production is at the limit of possibility
- Everybody happy



A tax on output of the firm

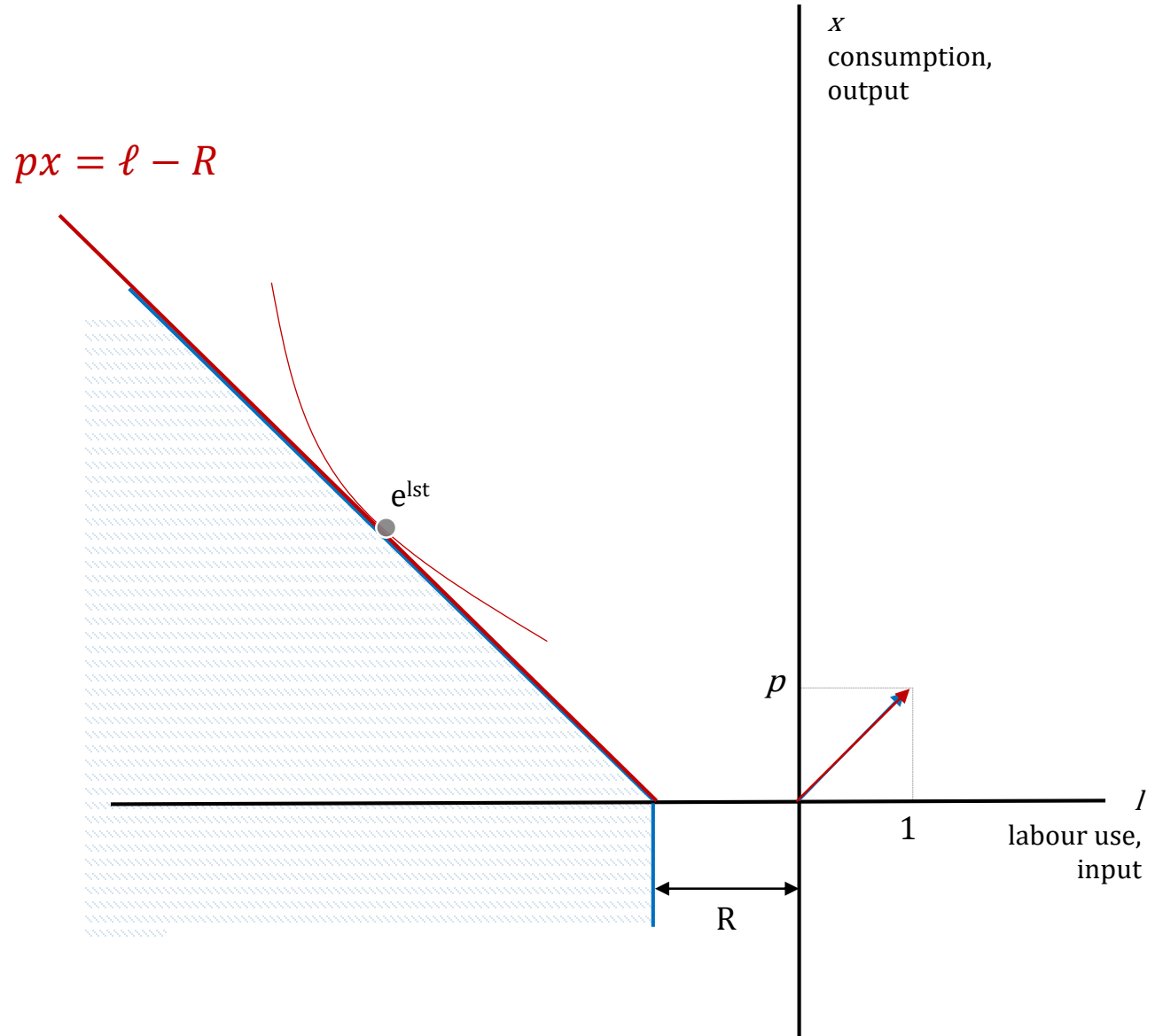
- Government needs to raise revenue R
- The first R units of labour input are commandeered by government
- We don't want to alter the relative prices faced by the producer (to maintain production efficiency)



Lump sum tax

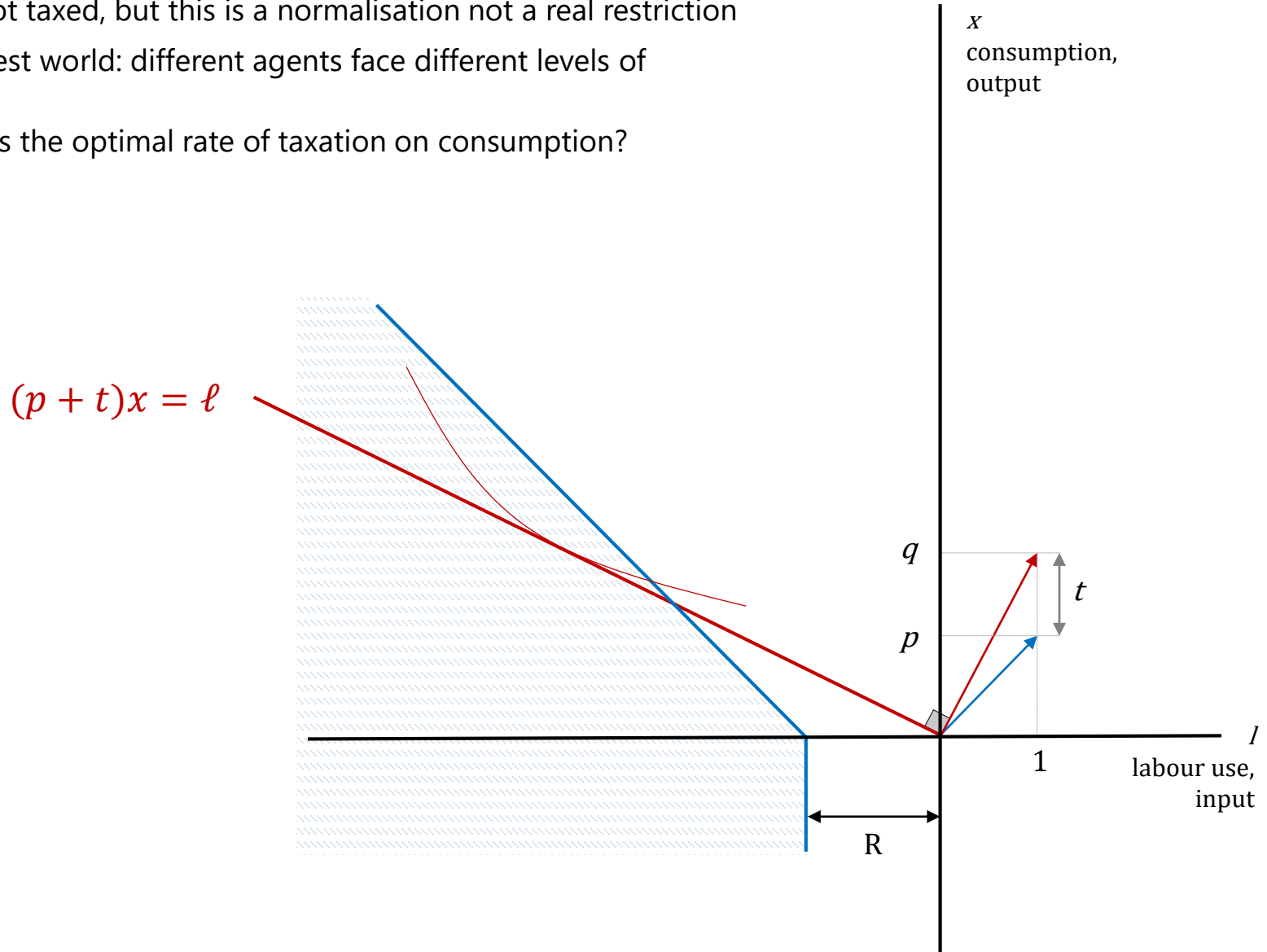
A lump sum tax would change the consumers income, without affecting the relative price of labour and consumption

But this is ruled out by assumption



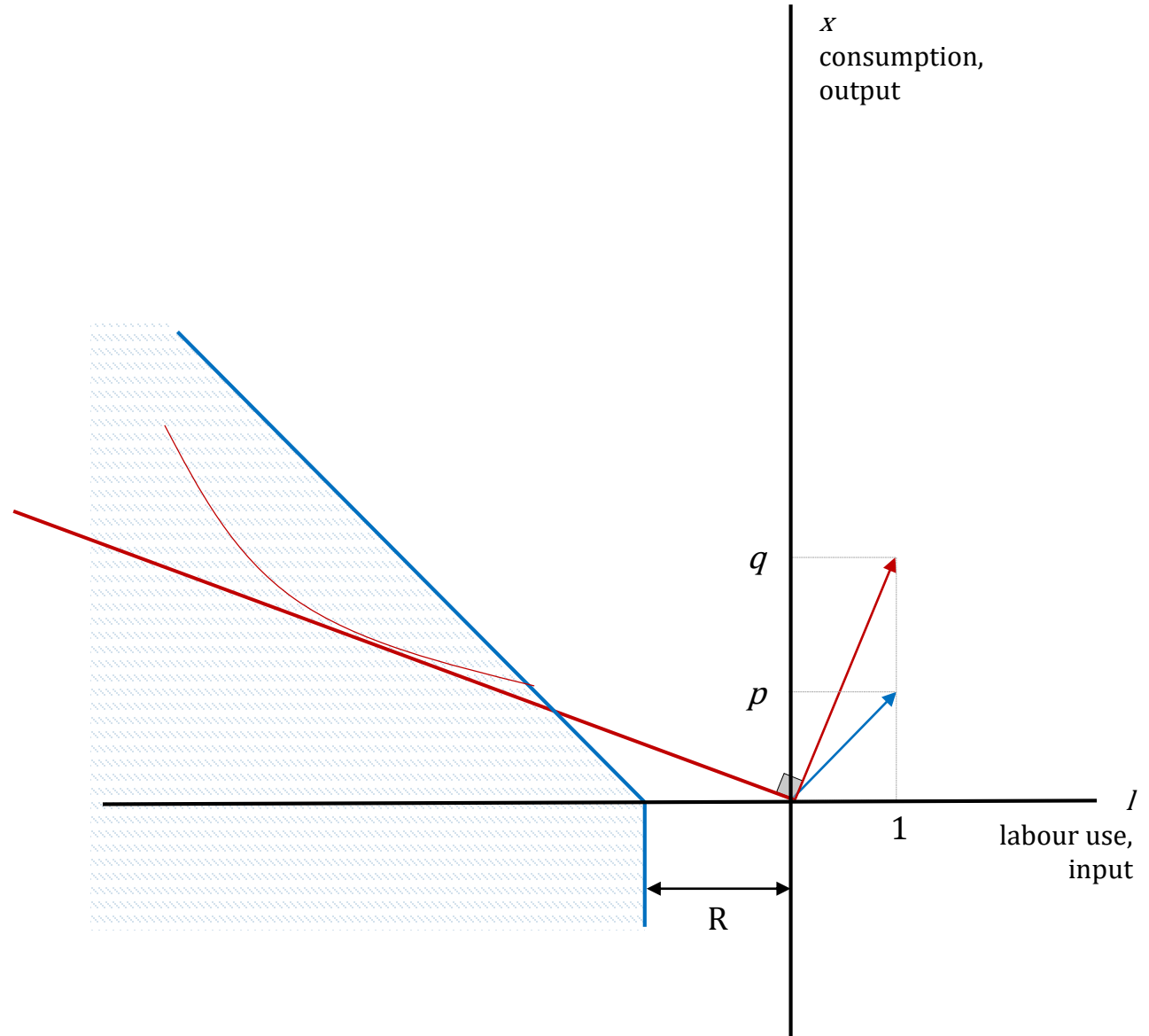
A tax on consumption

- This is achieved by driving a wedge between the cost of production (p) and the purchase price of output (q)
- Labour is not taxed, but this is a normalisation not a real restriction
- A second best world: different agents face different levels of taxation
- Now, what is the optimal rate of taxation on consumption?



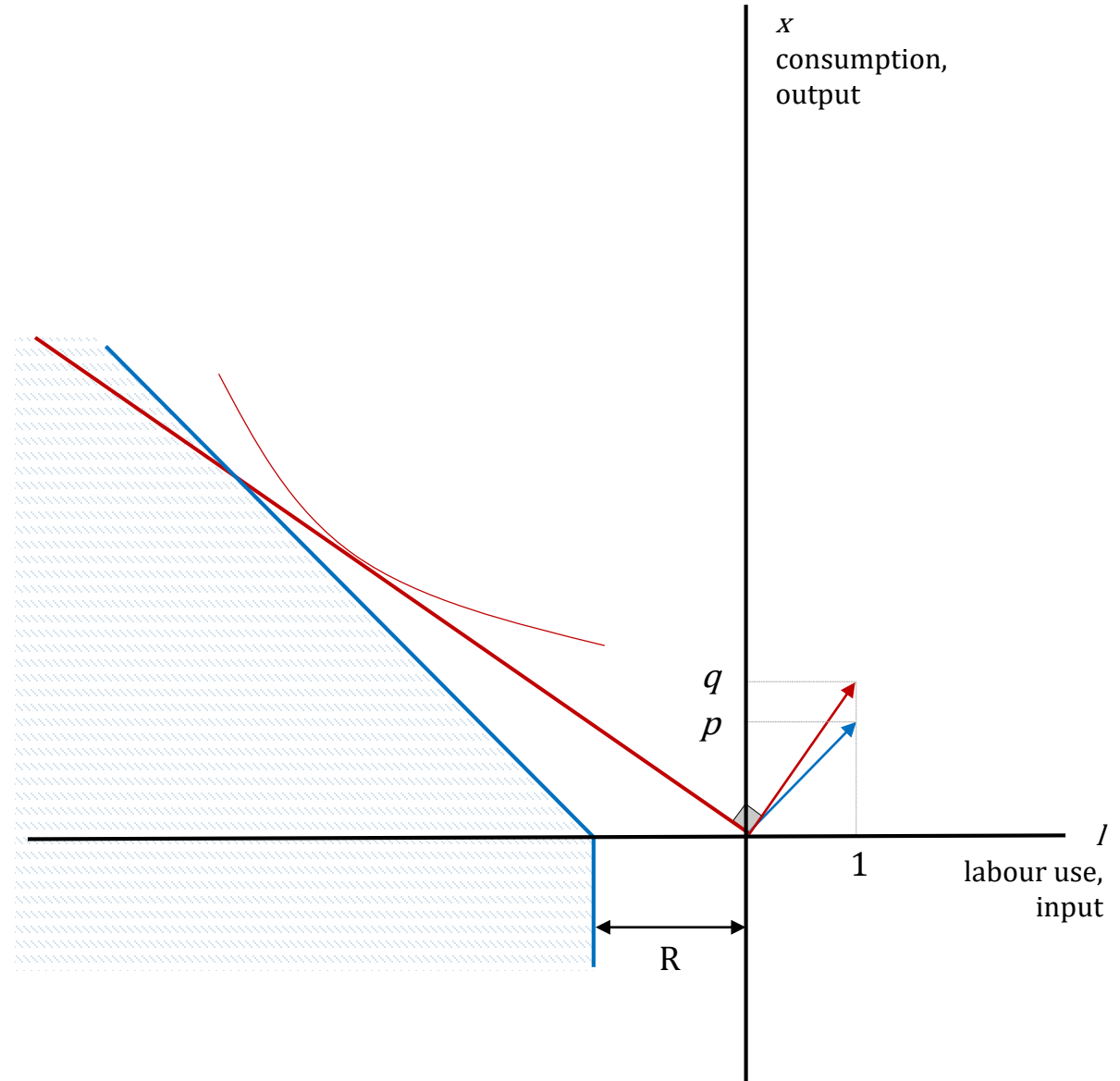
Two good economy - production

- What is the optimal rate of taxation on consumption? ($q-p$)



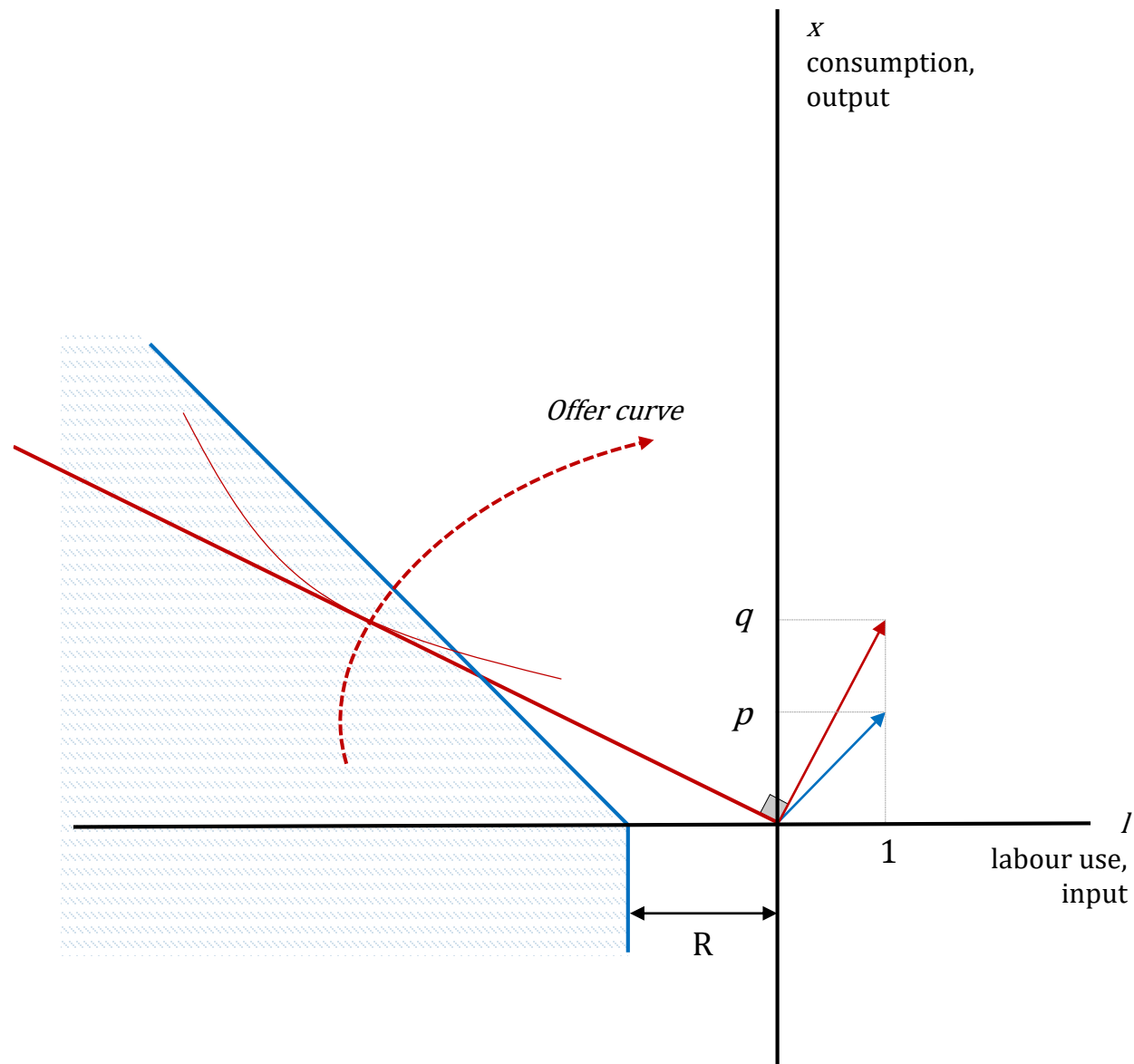
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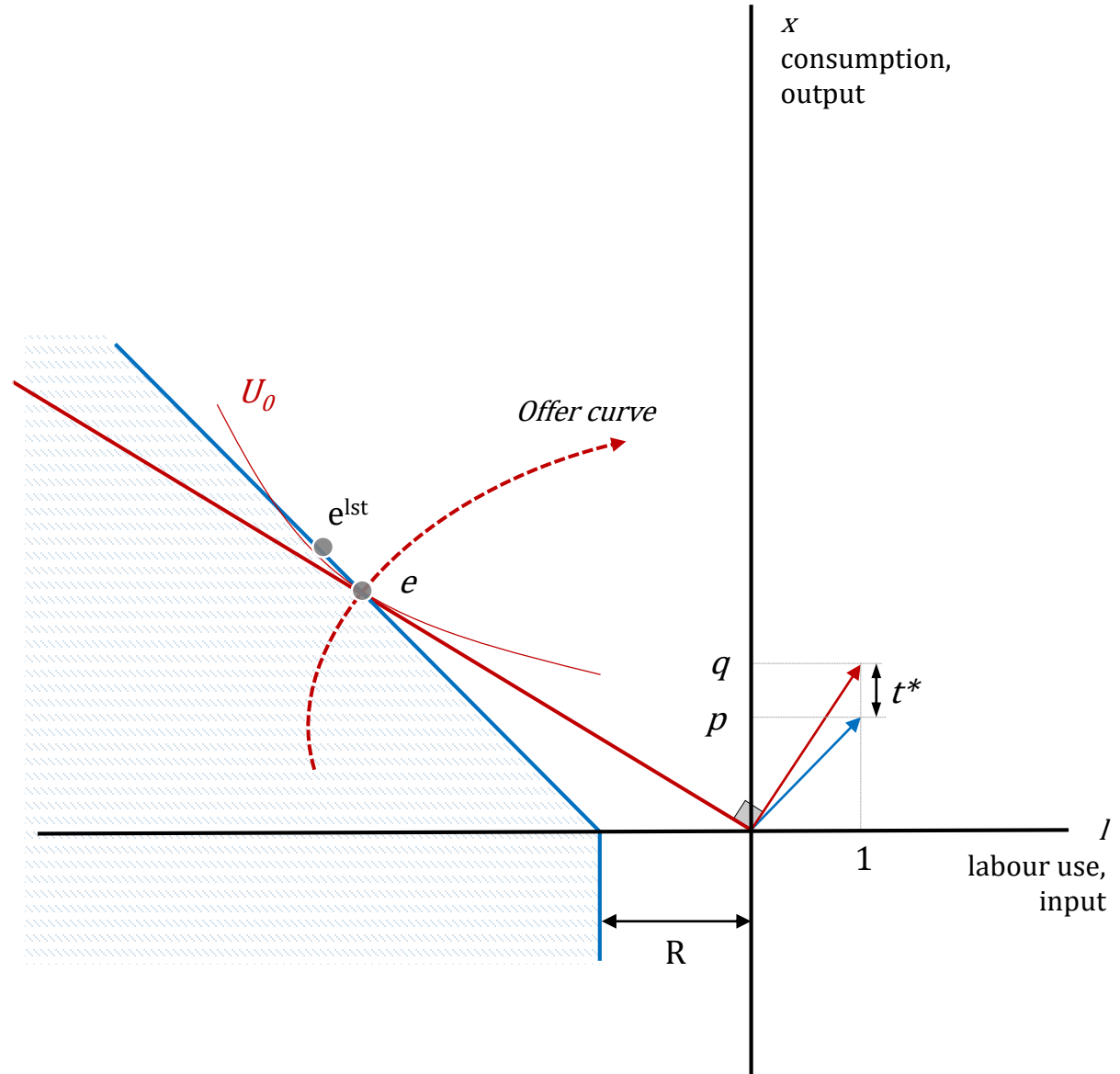
Two good economy - production

- **What is the optimal rate of taxation on consumption?**
($q-p$)
- The offer curve shows points consistent with utility maximization as commodity taxes vary q
- Each point on the offer curve is associated with a budget constraint that runs through the origin and an indifference curve tangent to that budget constraint.
- Utility increases along the offer curve



Optimal commodity taxation

- **What is the optimal rate of taxation on consumption?** ($q - p$)
- **The optimal commodity tax is determined at the highest point on the offer curve in the production set.**
- The optimal tax rate is t^* , which ensures that:
 - The consumer chooses point e
 - By construction this point raises $R = t^* x^*$, where x^* is consumption at point e
- The optimal rate e , is second best because e^{1st} raises the same revenue with a higher level of utility



- The producer is allowed to maximize leaving relative prices in tact.
- The economy must maximize output from the resource endowment
- Optimal commodity taxation should not disrupt production efficiency
($MRTS_{kl} = r/w$)
- The optimum with commodity taxation must be on the boundary of the production set
- Intermediate goods should not be taxed
- All distortions are focused on consumer choice.

- **Optimal taxation:** The set of taxes that results in the highest level of welfare while raising the revenue required by government
- Modelling approach
 - Firms maximize output on the production efficiency frontier
 - Constant returns to scale fixes producer prices, and ensures that any demand is met with supply
 - The problem is then divided into two problems
 - **The consumers problem:** Consumer left free to choose most preferred consumption plans, maximizes utility to derive the offer curve in the context of inevitable price distortions
 - **The government's problem:** achieve the revenue constraint while maximizing the consumers welfare by choosing the maximum point on the offer curve.
 - **The solution** is the determined at the highest point of the offer curve in the production set
 - An optimal allocation is chosen (e), and the **tax rate is inferred** from this point
- Commodity taxes are second best, relative to lump sum taxes (the level of welfare is always lower

- In this example we had only one consumer, so **equity** was not at issue
- We also had only **one commodity**: What is the solution if we have more than one commodity?
- Should all goods have the same rate or should taxes be related to the characteristics of the good

- Using only commodity taxes, maximize social welfare subject to raising required revenue
- Labour (input) produces commodities (output)
- Labour (x_0) is the *numeraire*, it is the only good not taxed ($t_0 = 0$)
- Two commodities (x_i where $i = 1, 2$)
- Independent demands (i.e. no cross-price effects) – this is a very strong simplifying assumption, which is relaxed in the more general model
- Commodities are produced competitively at constant returns to scale:
 - $p_i = MC = c_i$ where c_i units of labour needed to produce commodity i
 - Equilibrium prices are independent of the level of demand
 - In equilibrium supply will meet any level of demand.

- Producer price: $p_i = c_i$
- Consumer price: $q_i = p_i + t_i$
- Governments revenue requirement: $R = t_1x_1 + t_2x_2$

- The consumers buys two taxed goods and sells labour

$$\max U(x_0, x_1, x_2) \text{ s. t. } q_1 x_1 + q_2 x_2 = x_0$$

$$\mathcal{L} = U(x_0, x_1, x_2) + \alpha(x_0 - q_1 x_1 - q_2 x_2)$$

- First order conditions

$$\frac{\partial \mathcal{L}}{\partial x_0} = U'(x_0) + \alpha = 0$$

$$U'(x_0) = -\alpha$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = U'(x_1) - \alpha q_1 = 0$$

$$U'(x_1) = \alpha q_1$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = U'(x_2) - \alpha q_2 = 0$$

$$U'(x_2) = \alpha q_2$$

- Where α is the marginal utility of income
- NB the consumer does not internalize (is not affected by) government's budget

- Government's budget constraint is $R = t_1x_1 + t_2x_2$
- Since $t_i = q_i - p_i$ we can rewrite this as $q_1x_1 + q_2x_2 = R + p_1x_1 + p_2x_2$
- The government must choose an optimal tax rate that maximizes the consumer's utility subject to the government's revenue constraint.

$$\max_{\{x_1, x_2\}} \mathcal{L} = U(x_0, x_1, x_2) + \lambda(q_1x_1 + q_2x_2 - R - p_1x_1 - p_2x_2)$$

- NB
 - the labour supply is endogenously determined through the consumer's maximization, so we can substitute x_0 for $x_0 = q_1x_1 + q_2x_2$
 - The consumer's choices of x_i feed directly to the governments revenue constraint (this is the source of deadweight loss)
 - Inverse demand function $q_i = q_i(x_i)$
(since demand is independent; otherwise its too complicated)
 - Which means that: $x_0 = q_1(x_1) \cdot x_1 + q_1(x_2) \cdot x_2$
- So the first order condition (for commodity 1) becomes

$$\frac{\partial \mathcal{L}}{\partial x_1} = U'(x_1) + U'(x_0) \left[q_1 + x_1 \frac{\partial q_1}{\partial x_1} \right] + \lambda \left[q_1 + x_1 \frac{\partial q_1}{\partial x_1} - p_1 \right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = U'(x_1) + U'(x_0) \left[q_1 + x_1 \frac{\partial q_1}{\partial x_1} \right] + \lambda \left[q_1 + x_1 \frac{\partial q_1}{\partial x_1} - p_1 \right] = 0$$

▪ But from the consumer's optimisation we have: $U'(x_1) = \alpha q_1$ $U'(x_0) = -\alpha$

▪ And remembering that $t_i = q_i - p_i$

▪ We can rearrange and simplify to get

$$-\alpha \left(x_1 \frac{\delta q_1}{\delta x_1} \right) + \lambda t_1 + \lambda \left(x_1 \frac{\delta q_1}{\delta x_1} \right) = 0$$

▪ Now, with respect to the post-tax price, the definition of elasticity is

$$\varepsilon^d \equiv \frac{q}{x} \frac{\partial x}{\partial q} \qquad \frac{1}{\varepsilon^d} = \frac{x}{q} \frac{\partial q}{\partial x}$$

▪ So we can substitute to get

$$-\alpha \left(\frac{q_1}{\varepsilon_1^d} \right) + \lambda t_i + \lambda \left(\frac{q_1}{\varepsilon_1^d} \right) = 0$$

▪ And further rearrange and simplify to conclude and state the inverse elasticity rule

$$\frac{t_1}{p_1 + t_1} = - \left[\frac{\lambda - \alpha}{\lambda} \right] \frac{1}{\varepsilon_1^d}$$

$$\frac{t_i}{p_i + t_i} = - \left[\frac{\lambda - \alpha}{\lambda} \right] \frac{1}{\varepsilon_i^d}$$

- α is the consumers marginal utility of income
- λ is the utility cost of government's marginal revenue
- Since λ includes deadweight loss $\lambda > \alpha$
- $\varepsilon^d < 0$ so the tax rate is positive.

- The proportional rate of tax on good i should be inversely related to its price elasticity of demand
- The constant of proportionality $\left[\frac{\lambda - \alpha}{\lambda} \right]$ is the same for all goods
- The goods with inelastic demand have lower deadweight loss, and must therefore attract a higher rate of taxation.
- The assumption of independent demand is relaxed to give a more general version
- Very efficient, but is it equitable?

$$\frac{t_i}{q_i} = k \left(\frac{1}{\varepsilon_u^d} + \frac{1}{\varepsilon^s} \right)$$

Where: k depends on revenue requirement

t tax per unit

q after tax price

ε^d compensated elasticity of demand

ε^s elasticity of supply

The main conclusion as $\varepsilon^s \rightarrow \infty$ $t/q \rightarrow k / \varepsilon^d$

Optimal income taxation

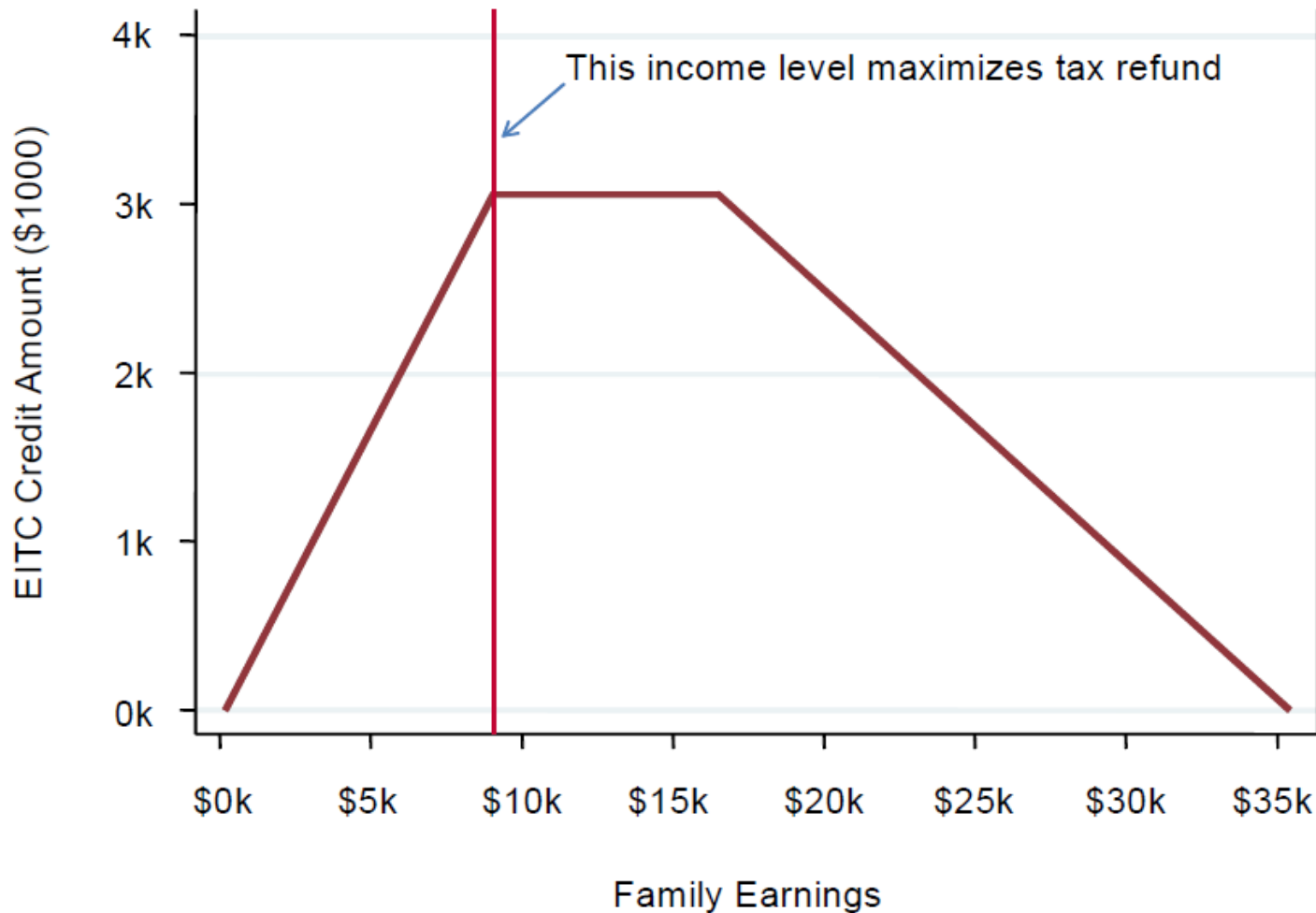
Personal income tax brackets in South Africa

Table 4.4 Personal income tax rates and bracket adjustments

2018/19		2019/20	
Taxable income (R)	Rates of tax	Taxable income (R)	Rates of tax
R0 - R195 850	18% of each R1	R0 - R195 850	18% of each R1
R195 851 - R305 850	R35 253 + 26% of the amount above R195 850	R195 851 - R305 850	R35 253 + 26% of the amount above R195 850
R305 851 - R423 300	R63 853 + 31% of the amount above R305 850	R305 851 - R423 300	R63 853 + 31% of the amount above R305 850
R423 301 - R555 600	R100 263 + 36% of the amount above R423 300	R423 301 - R555 600	R100 263 + 36% of the amount above R423 300
R555 601 - R708 310	R147 891 + 39% of the amount above R555 600	R555 601 - R708 310	R147 891 + 39% of the amount above R555 600
R708 311 - R1 500 000	R207 448 + 41% of the amount above R708 310	R708 311 - R1 500 000	R207 448 + 41% of the amount above R708 310
R1 500 001 and above	R532 041 + 45% of the amount above R1 500 000	R1 500 001 and above	R532 041 + 45% of the amount above R1 500 000
Rebates		Rebates	
Primary	R14 067	Primary	R14 220
Secondary	R7 713	Secondary	R7 794
Tertiary	R2 574	Tertiary	R2 601
Tax threshold		Tax threshold	
Below age 65	R78 150	Below age 65	R79 000
Age 65 and over	R121 000	Age 65 and over	R122 300
Age 75 and over	R135 300	Age 75 and over	R136 750

Source: National Treasury

Earned Income Tax Credit Schedule for Single Earners with One Child in 2008



- From an efficiency point of view we should fund expenditure with lump sum taxes
- Poll tax (efficient but highly regressive)
- If we are concerned with equity or redistribution we need individual specific lump sum tax related to intrinsic ability
- E.g. A height tax:
 - In a world where height was a perfect predictor of income, we could impose a lump sum tax on height
 - Height is observable
 - It is endowed and cannot be changed (leaving aside childhood nutrition etc)
- But in the real world
 - We cannot directly observe individual's ability
 - Revealing ability is not incentive compatible
 - Therefore we must tax economic outcomes such as income or consumption
 - When we tax outcomes we change incentives to attain those outcomes (e.g. reducing the income associated with labour may reduce the supply of labour)
- This leads to distortions
- And distortions lead to a trade off between equity and efficiency

- Consideration of the optimal system must be flexible enough to consider a variety of forms
 - Whereas commodity taxation is typically restricted to τX or $(p+t)X$
 - Income taxation schemes take the form of $t(X)$
- Take account of efficiency objectives (labour supply)
 - Positive question: how does income taxation affect labour supply
- Take account of redistributive objectives (equity)
 - Normative question: how should the income tax structure be determined
- Optimal tax system: maximizes social welfare (equity and efficiency) subject to raising governments revenue requirement

Taxation and labour supply model

- Workers vary hours of work/leisure to optimize income/consumption subject to a budget constraint
- Assuming there are no savings, so consumption is identical to income

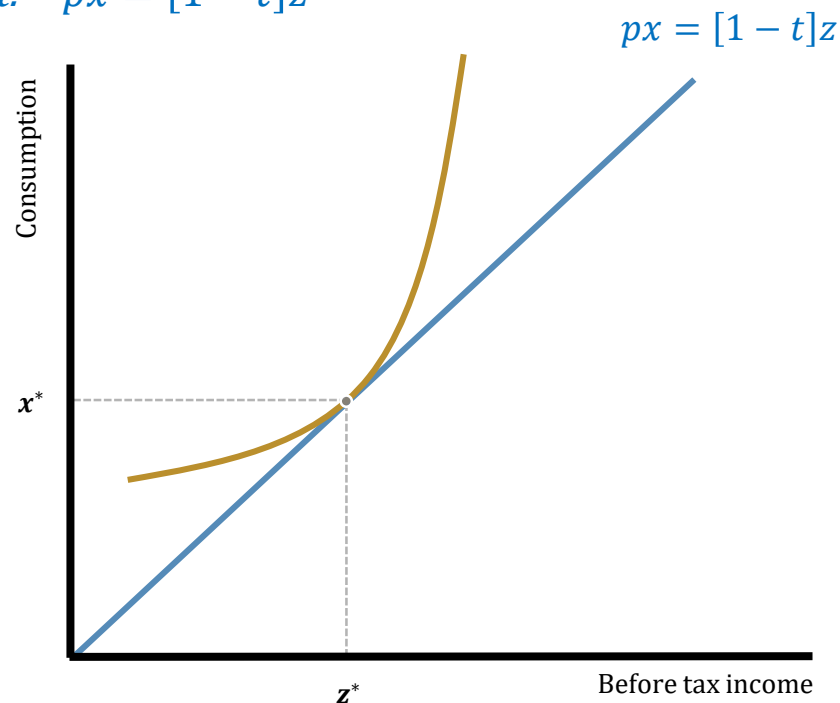
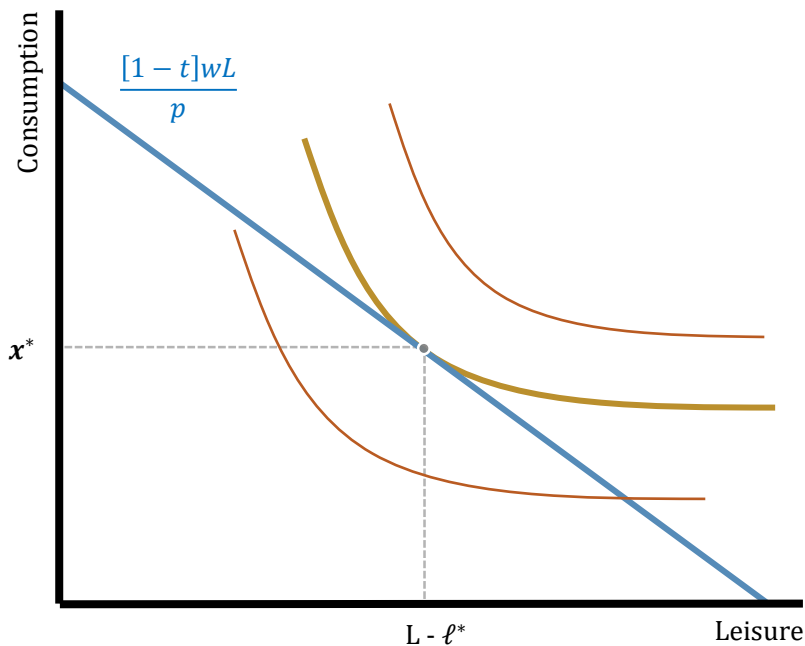
$$U = U(x, L - \ell) = U(x, \ell)$$

$$\text{s.t. } px = [1 - t]w\ell$$

$$z = w\ell \quad \ell = \frac{z}{w}$$

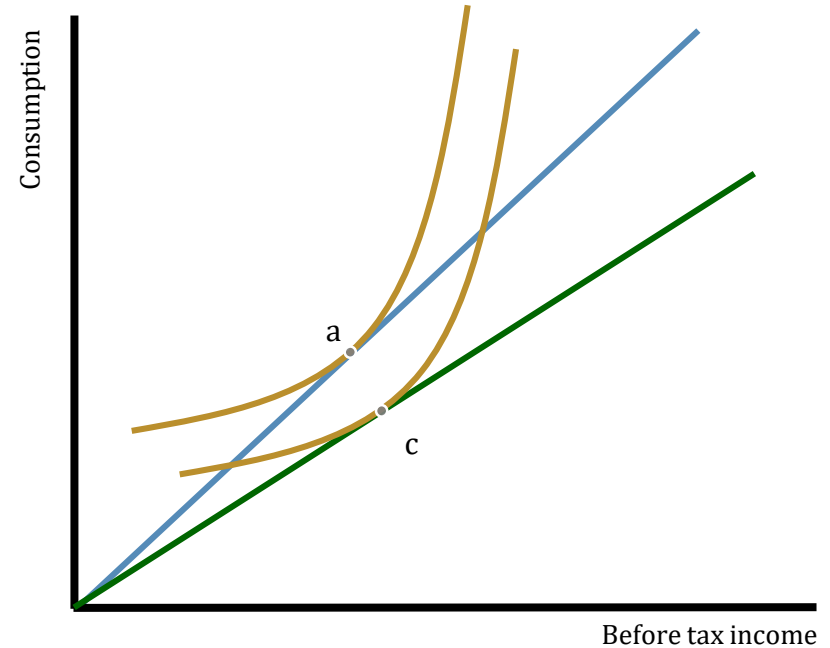
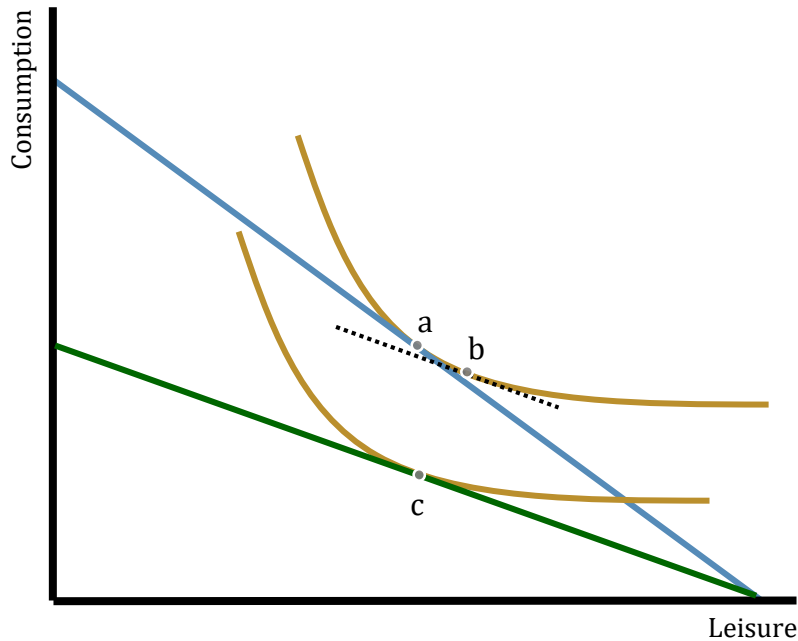
$$U = U\left(x, \frac{z}{w}\right)$$

$$\text{s.t. } px = [1 - t]z$$



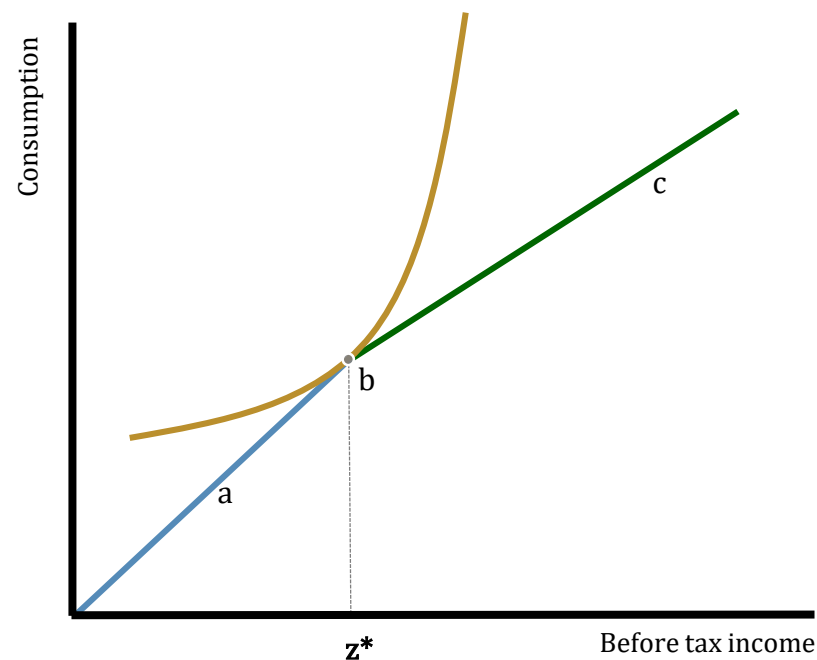
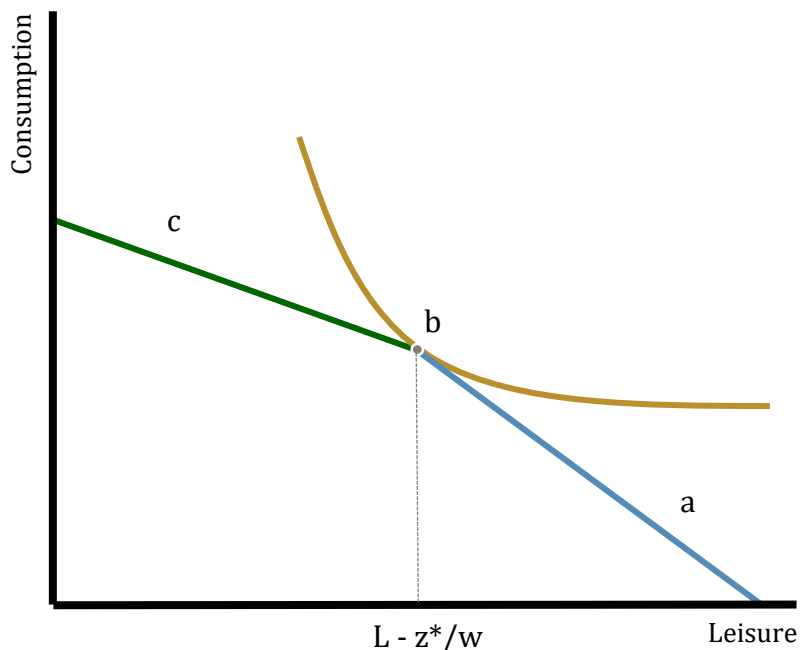
Tax increase and labour supply (intensive margin)

- What is the impact of a tax increase on hours worked?
 - Substitution effect is always the right sign
 - Income effect can move either way
- The outcome is therefore indeterminate (without specification)
- NB:
 - Movement from a to b is the compensated effect
 - Movement from a to c is the uncompensated effect



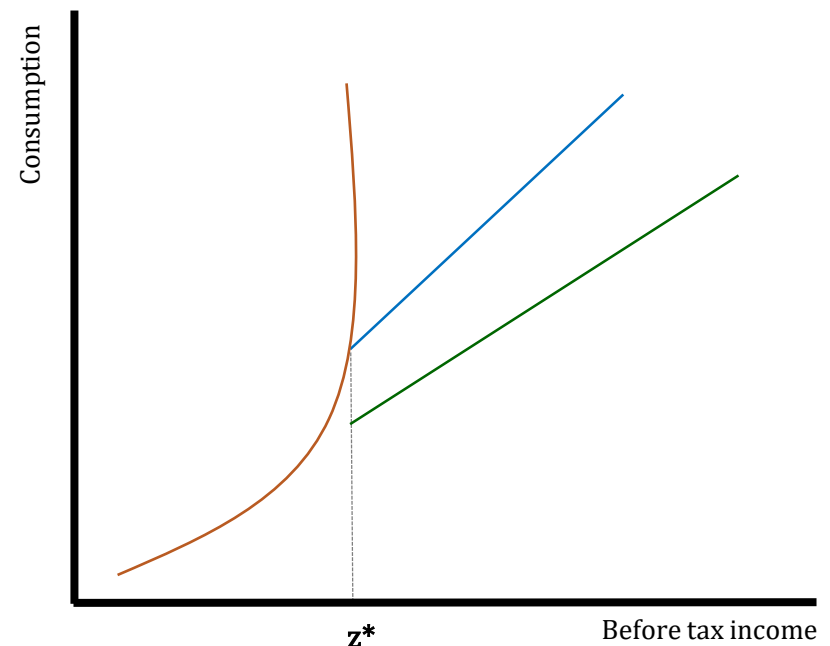
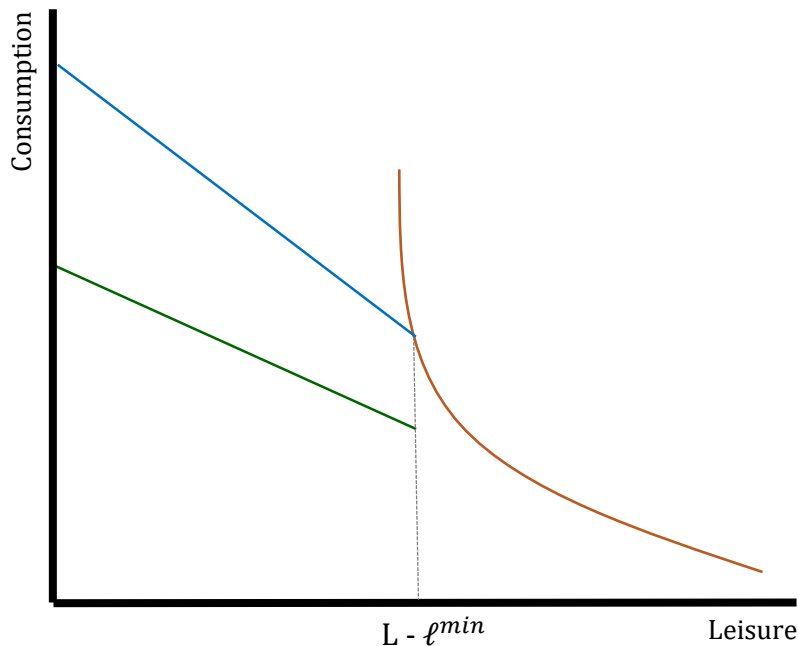
Tax threshold

- Tax is now paid only on earnings above z^*
- At point a no tax is paid, at point c tax is paid
- Many consumers will “bunch” at b
 - At b an extra unit of labour receives net-of-tax return of $[1-t]w$, whereas the previous marginal unit received w .
 - If a consumer is optimizing at b, a change in the marginal tax rate might not change the optimal point
 - While there will be no substitution effects, income effects still apply
- Interior solutions (a, c) and corner solution (b)
- Most tax systems have several threshold points



Labour force participation (extensive margin)

- What if the number of working hours cannot be varied continuously to arrive at preferred income
- In practice, the hours of work are fixed: there is a minimum working day, regulated by firms and firm-union agreements
- Workers must “choose” whether or not to work or not (useful to think of non-working spouse)
- A worker who was indifferent between working and not working is now committed to not working



- Theory suggests three major issues in the relationship between labour supply and taxation
 - Interior solution can go either way (income and substitution)
 - Corner solutions could make labour supply insensitive to tax changes
 - Participation decision can be sensitive to taxation
- Survey evidence
 - “Surveys on labor supply have normally arrived at the conclusion that changes in the tax rate have little effect on the labour supply decision”
 - Lawyers and accountants at the intensive margin
 - Weekly overtime
 - Suggestive of a vertical labour supply function
 - Income effect offsetting the substitution effect?
- Econometric estimation of the labour supply elasticities
 - Income effect always negative, an usually outweighs substitution effect
 - Men seem to have almost vertical/negative supply function
 - Women more elastic

Labour-supply elasticities

	<i>Married women</i>		<i>Married men</i>		<i>Single Mothers</i>	
	USA	UK	USA	UK	USA	UK
Uncompensated wage	0.45	0.43	-0.03	-0.23	0.53	0.76
Compensated wage	0.90	0.65	0.95	0.13	0.65	1.28
Income	-0.45	-0.22	-0.98	-0.36	-0.18	-0.52

Optimal income taxation Model setup

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- Two commodities: consumption (x) and labour (ℓ)
- $z(s) = s \cdot \ell(s)$ s measures the hourly output of the consumer, equal to the wage rate
- What is the optimal income tax function: $T(z)$
- $x = c(z) = z - T(z)$
- $c'(z) = 1 - T'(z)$

- Note that:

- On 45-degree line

- $c'(z) = 1$ so $T'(z) = 0$

- At point a

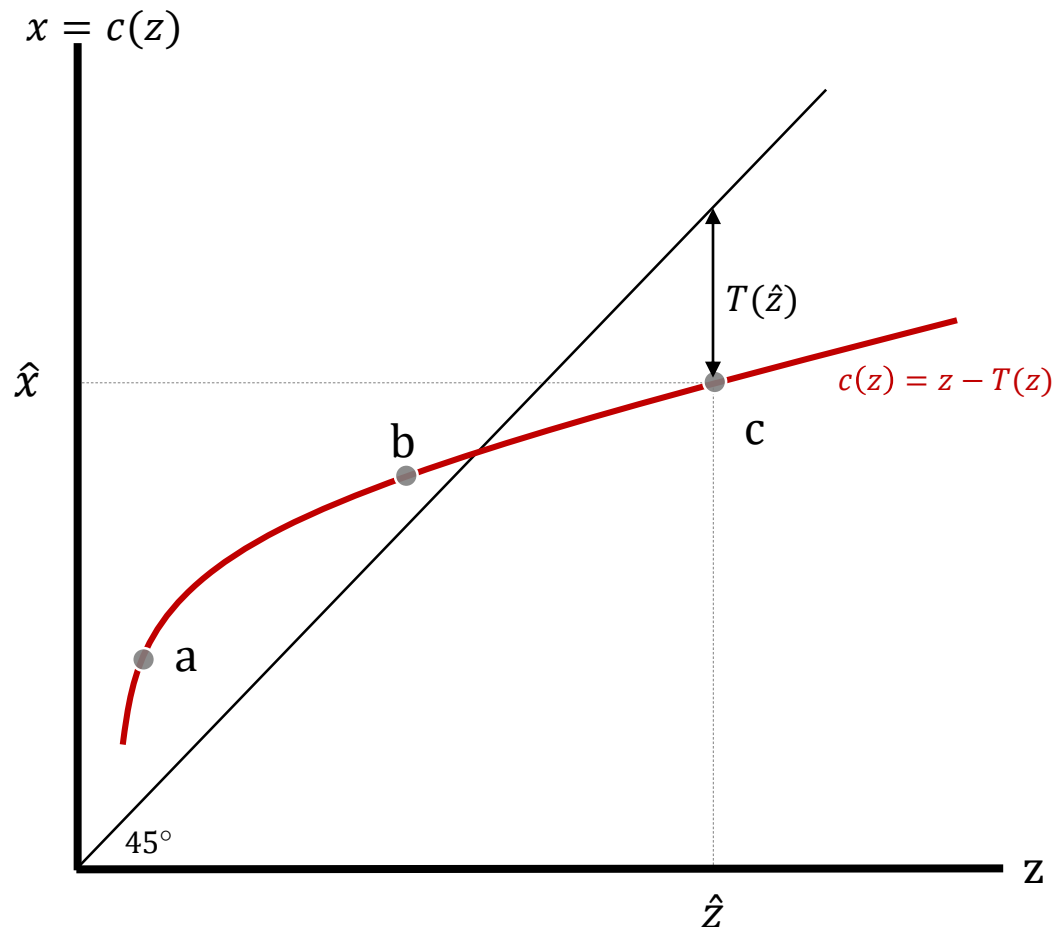
- $c'(z) > 1$ so $T'(z) < 0$

- At point b

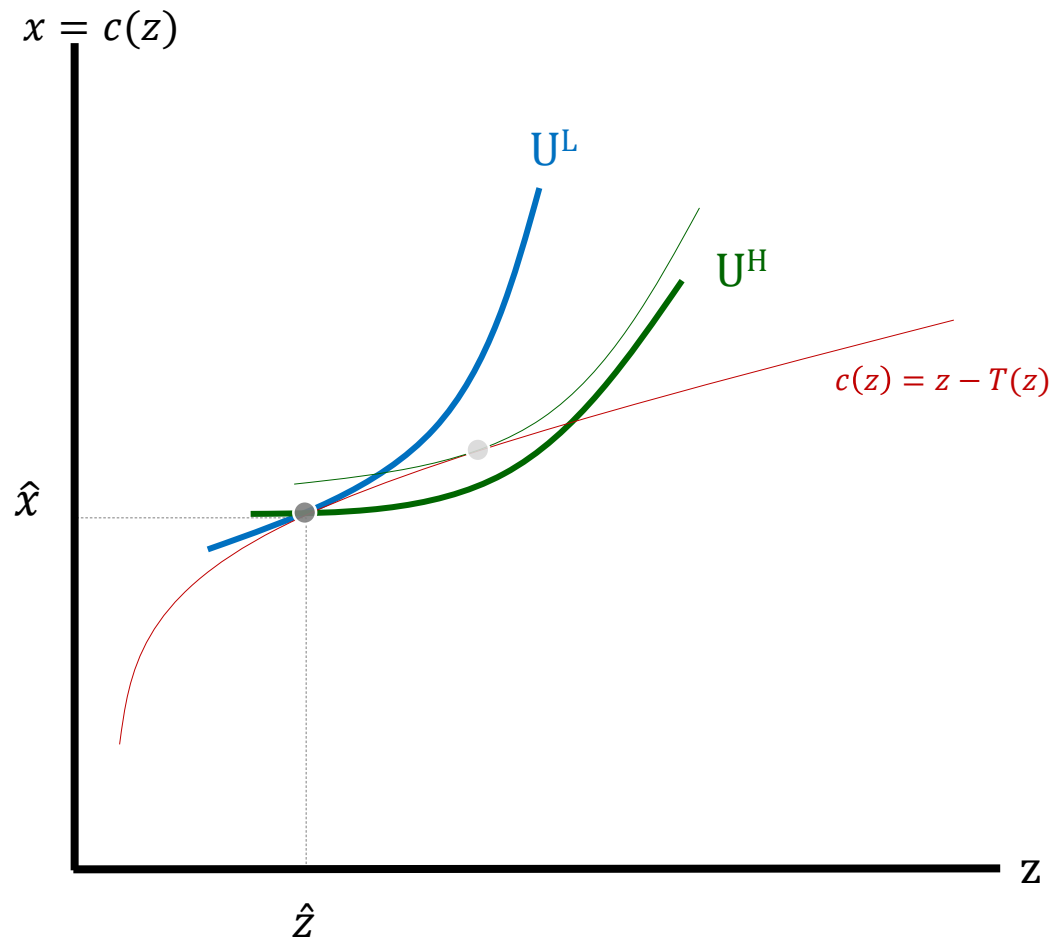
- $c'(z) < 1$ so $T'(z) > 0$

- A negative (marginal) tax rate implies a marginal subsidy

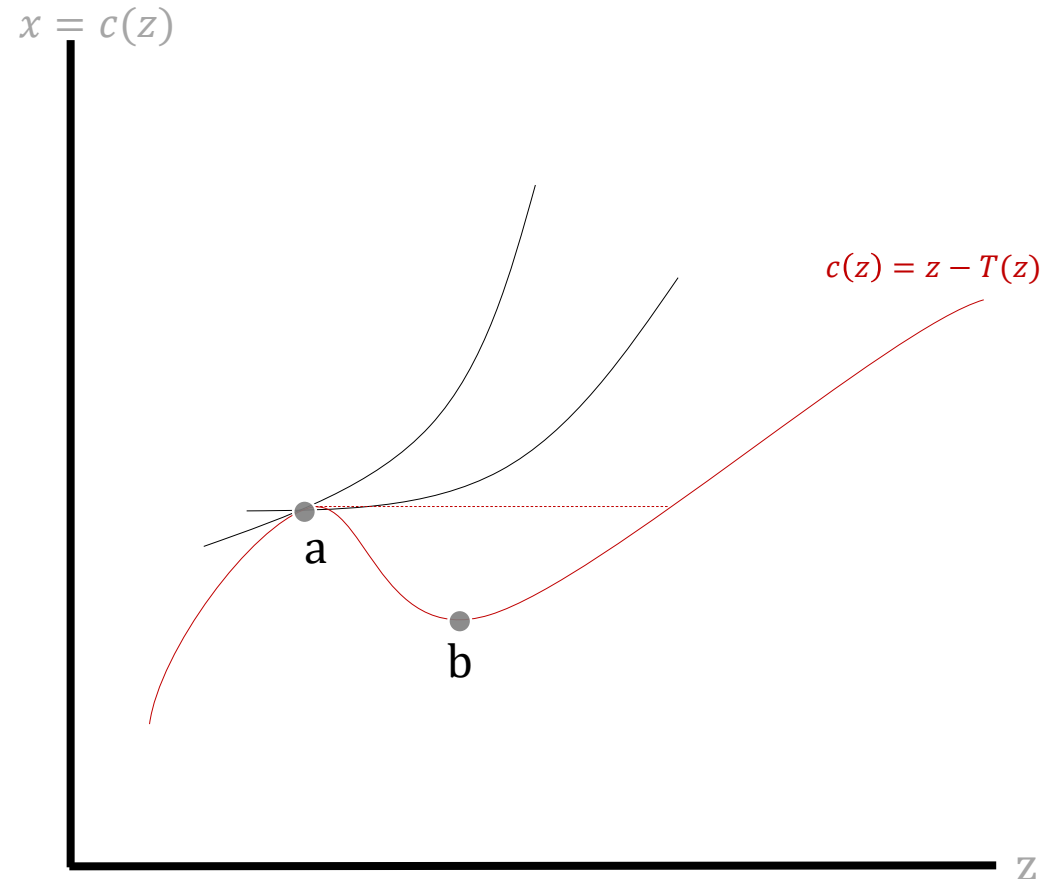
- The after tax wage for additional work will be greater than the before tax wage (e.g. EITC)



- All consumers have the same utility function: $U = U\left(x, \frac{z}{s}\right)$
- But different levels of skill: s^L and s^H
- A high-skill consumer takes less labour time to achieve a given level of income than a low skill consumer
- So, at any income consumption pair $\{\hat{x}, \hat{z}\}$ the indifference curve of the high-skill consumer is flatter (*agent monotonicity*)
- Taxation is on income choice $[T(z)]$ not on skill (s) so both face the same consumption function
- High-skill consumer will always (choose to) earn strictly more than low-skill consumer



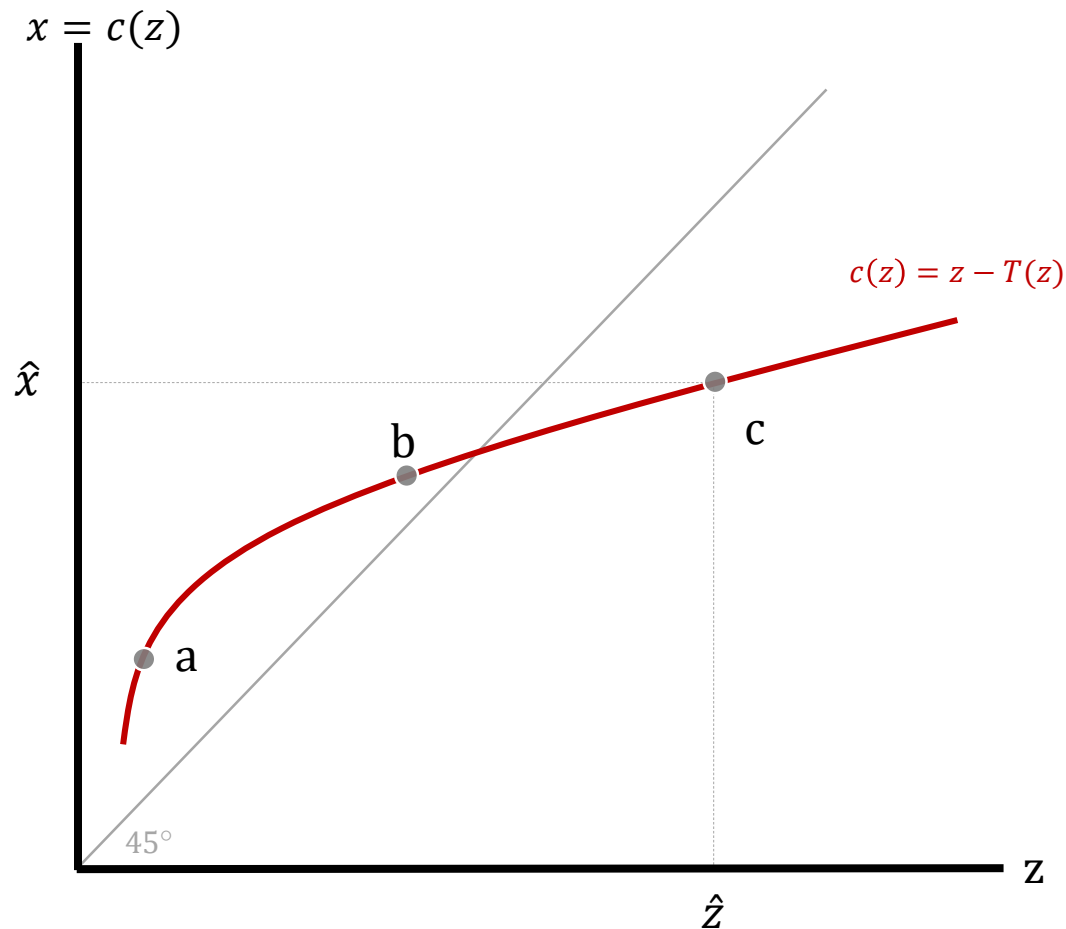
- Between a and b, increased income (work effort) leads to lower consumption
- Given preferences as stated, no consumer would choose a point between a and b
- $c(z) = z - T(z) \Rightarrow c'(z) = 1 - T'(z)$
- Therefore: $c'(z) \geq 0$
- Which implies: $T'(z) \leq 1$
- **So the optimal marginal tax rate must be strictly less than 100%**



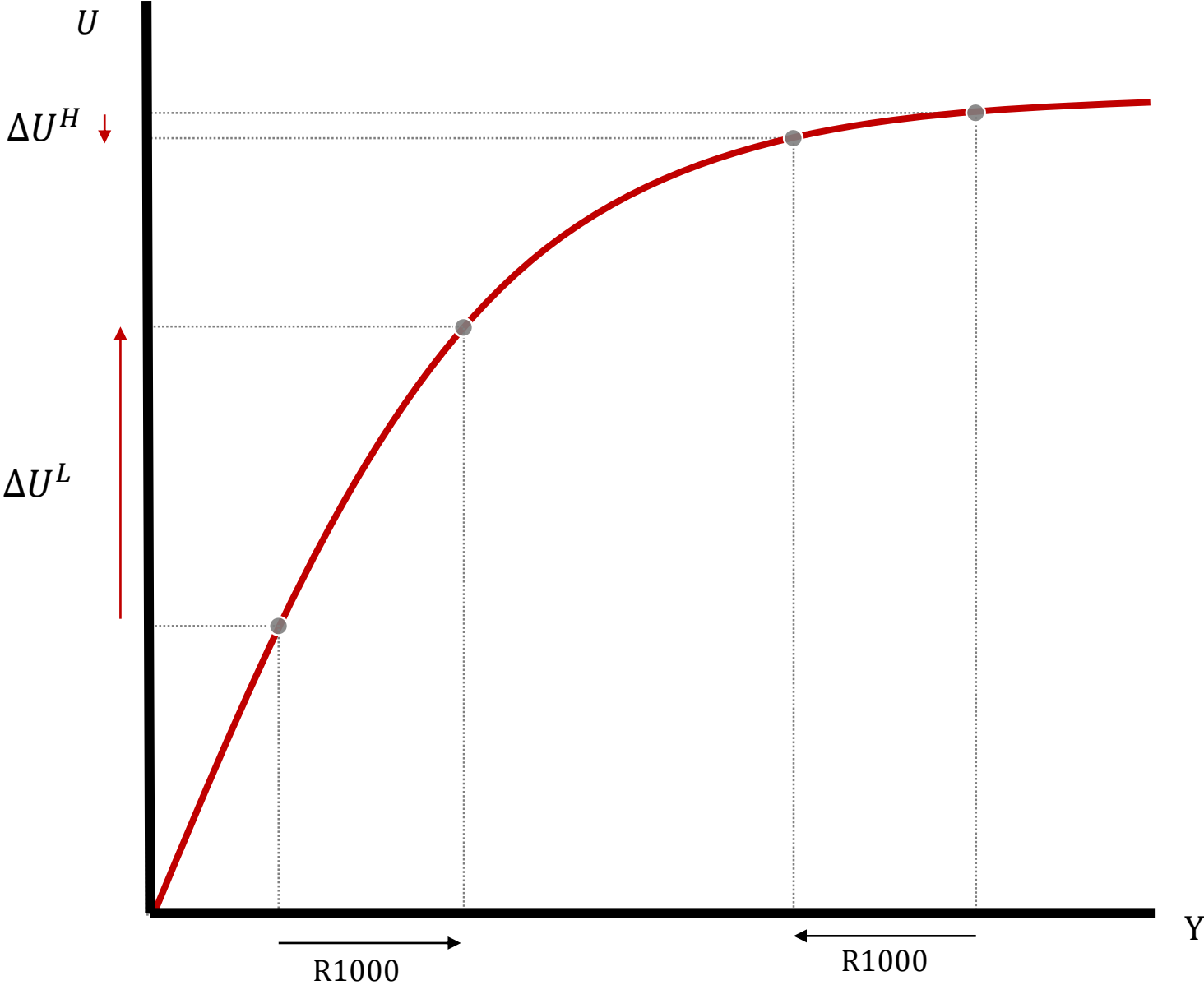
Optimal income taxation Lower limit?

- $c(z) = z - T(z) \Rightarrow c'(z) = 1 - T'(z)$
- Note: On 45-degree line: $c'(z) = 1$ so $T'(z) = 0$ marginal tax rate is zero
- At point a $c'(z) > 1$ so $T'(z) < 0$ marginal tax rate is negative
- At point b $c'(z) < 1$ so $T'(z) > 0$ marginal tax rate is positive

- We showed that (optimally) the marginal tax rate should be less than 1
- **But is there an optimal lower limit for $T'(z)$?**
- To answer we must say something about the welfare function and equity

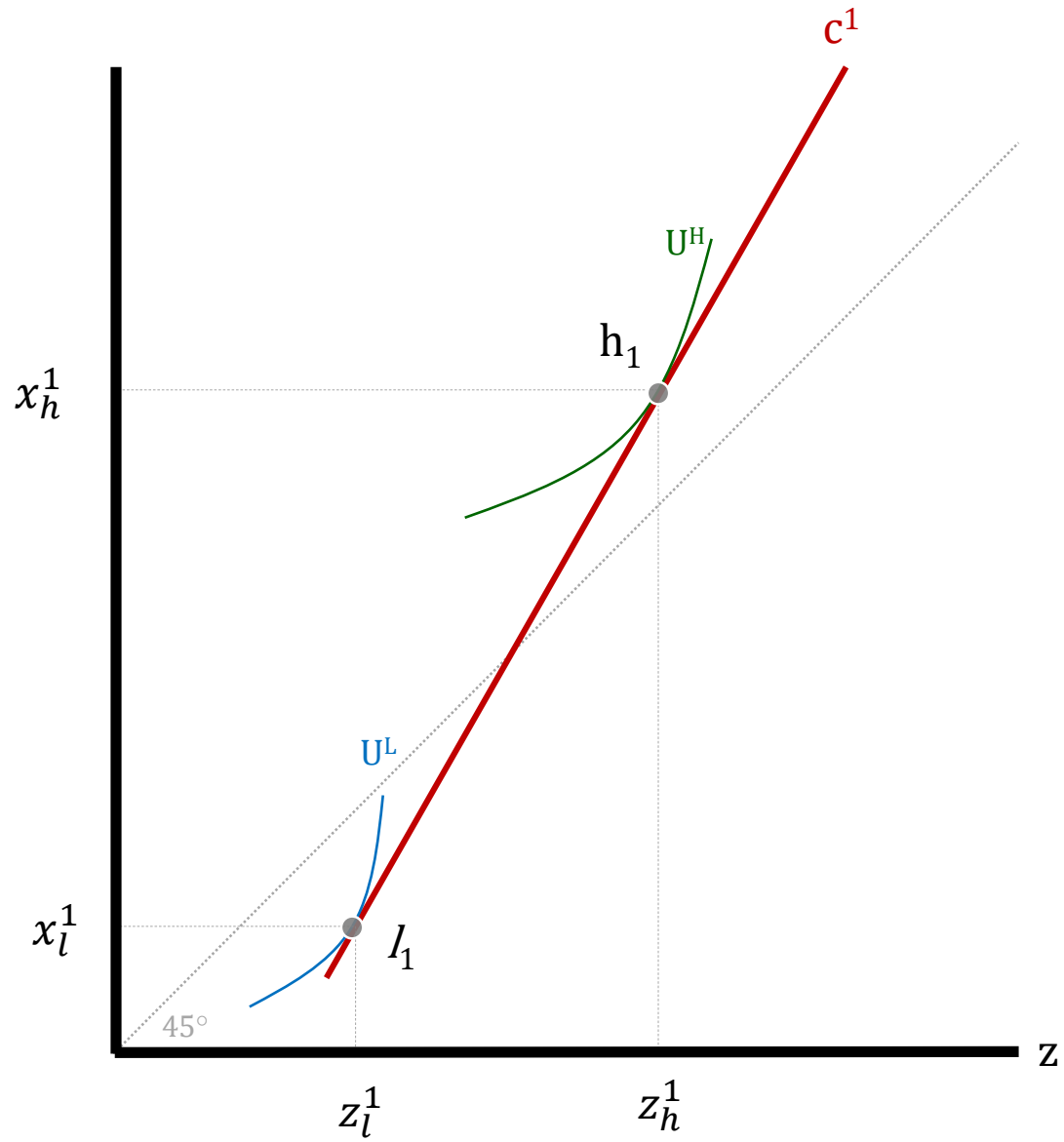


Utilitarian egalitarianism

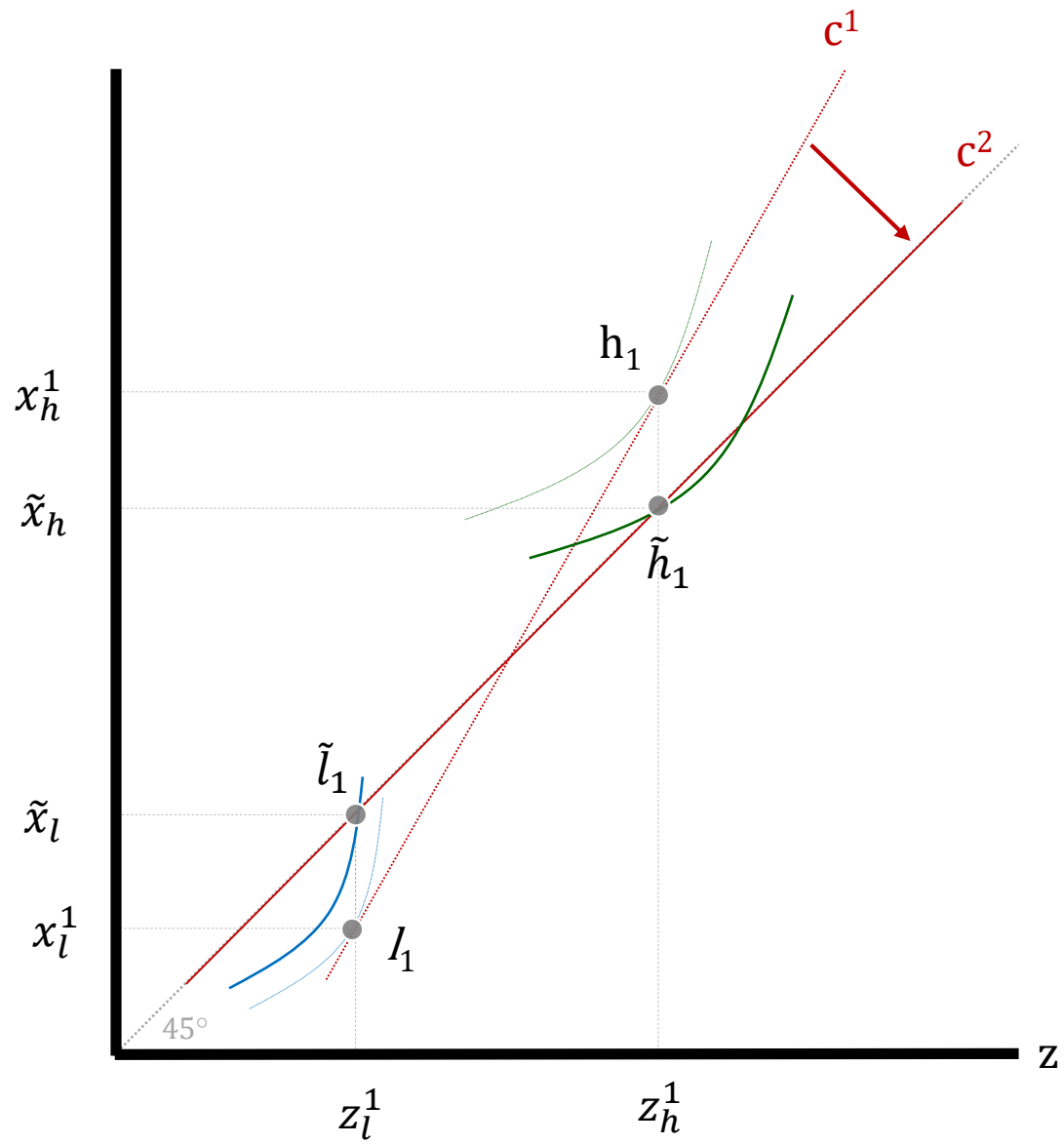


Optimal income taxation Can marginal tax rate be negative?

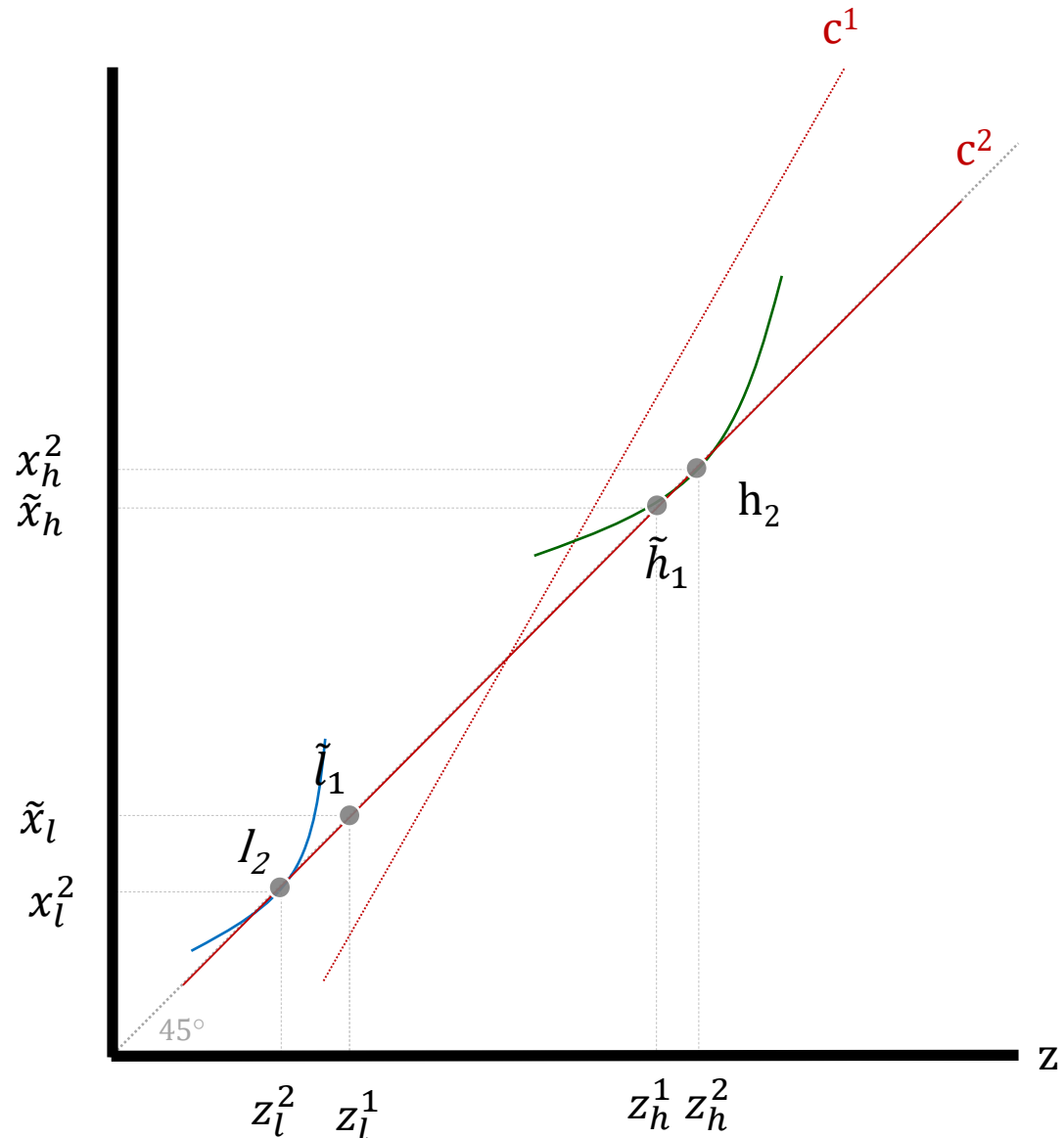
- Utilitarian welfare function
- Two types of consumers – low skill and high skill
- The $c^1(z)$ has a gradient greater than 1



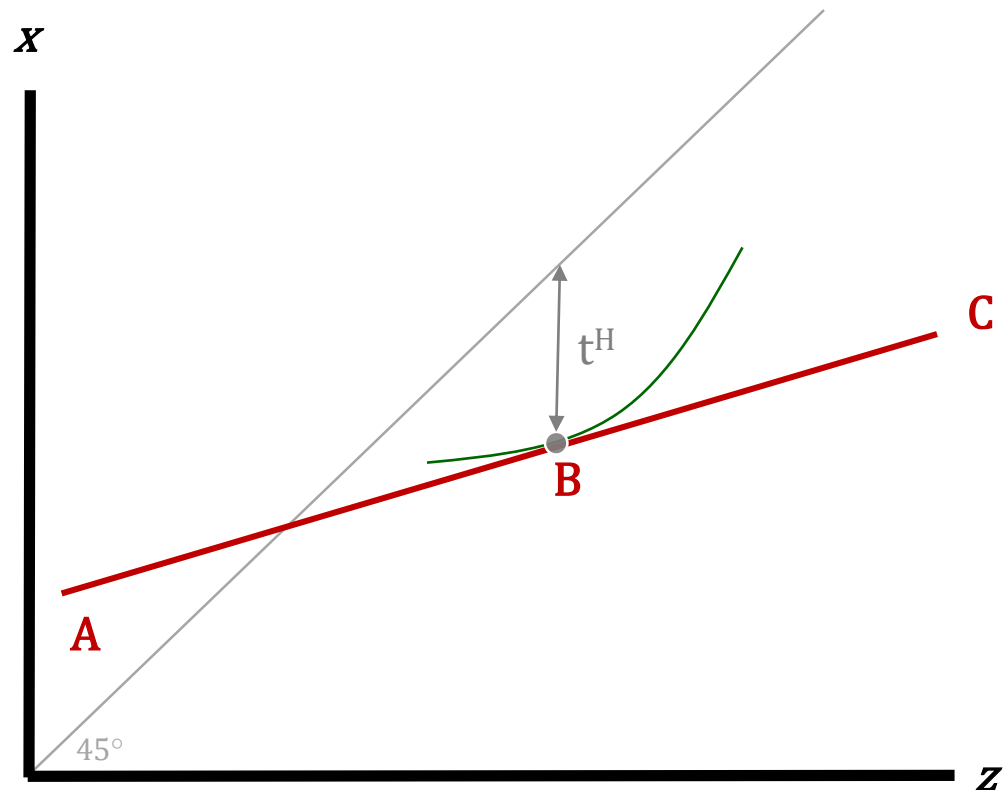
- The $c^1(z)$ has a gradient greater than 1
- The $c^2(z)$ gradient is 1
- Keeping income fixed the new consumption function changes consumption by the same amount – positive for the low skill, negative for the high skill
- Govt revenue has not changed (aggregate income and consumption are the same)
- But aggregate welfare has improved (because dU/dc is greater for the lower income individual)



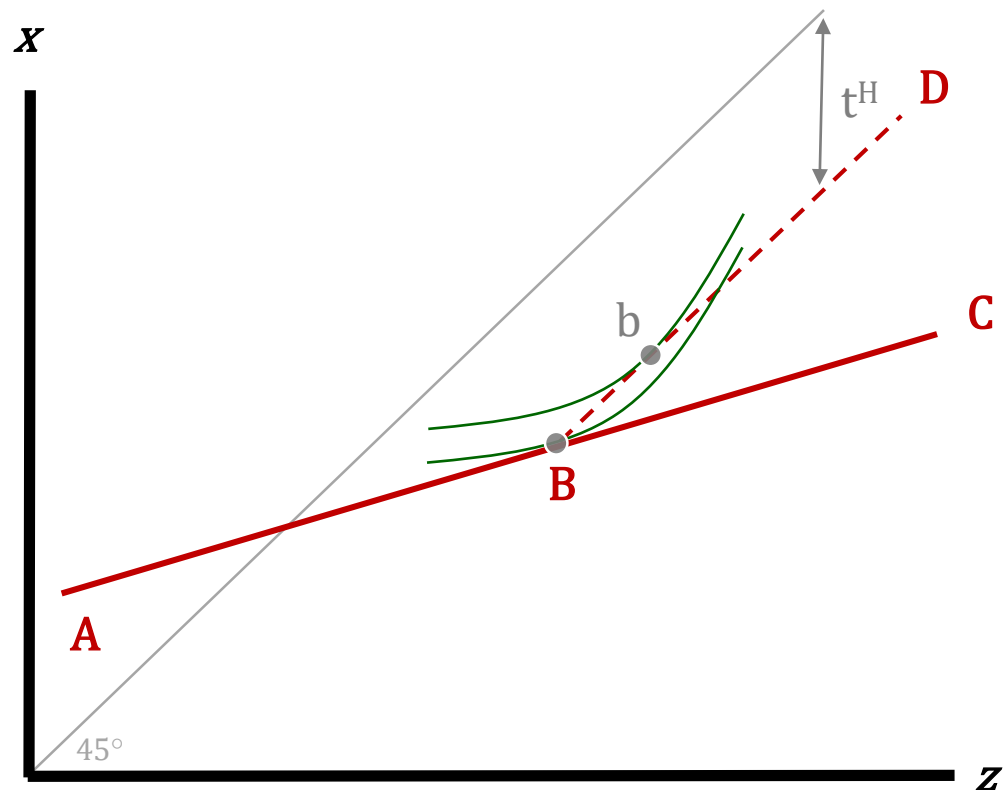
- Now let consumers optimize
- Relative to \tilde{l}_1 and \tilde{h}_1 utility is improved for both
- Government has not lost revenue as the movements are along a zero marginal tax rate curve
- So, with the same government revenue, c^2 must have higher social welfare than c^1
- So c^1 , with a negative marginal tax rate cannot be optimal
- **An optimal tax rate must be non-negative**
- $T'(z) \geq 0$
- Note: we have not made any conclusion on the average tax rate ($T(z)/z$) which could still be negative



- What marginal tax rate should we impose on the highest skill people?
- Lets post ABC as a candidate for optimality
- At point B the gradient of the function is less than 1, due to a positive marginal tax rate
- The high income consumer pays over t^H to the government



- Along alternative tax schedule ABD, the line BD is parallel to the 45 degree line (i.e. with zero marginal tax rate)
- The consumer moves to point b
- The government still gets t^H
- There is a pareto improvement (nobody is worse off as a result of the welfare improvement)
- **The optimal marginal tax rate for the highest skill person is zero**



- Optimal tax theory generates three key results:
 - The highest marginal rate must be strictly less than 1
 - Marginal tax rates should always be greater than zero
 - The marginal rate on top income earners should be zero
- Only the first conclusion is observed in practice (and its not a very interesting result)
- The other two results are honoured in the breach:
 - The EITC has a negative marginal rate
 - Top marginal rates are the highest
- And the overall conclusion that optimal tax rates should be between zero and one does not provide much practical guidance.

